Multiscale dynamic fracture behavior of the carbon nanotube reinforced concrete under impact loading

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ABSTRACT

In this paper, the impact behavior of the carbon nanotube reinforced concrete is investigated through the multiscale simulation. At the nano scale, properties of carbon nanotubes are determined through the molecular dynamics simulation. Afterwards, a finite element based hydration model of the cement is adopted to disperse the CNTs on the surface of the cement. At the meso scale, the concrete is simulated by considering all three phases including, cement, aggregates and interfacial transition zone (ITZ), to obtain a homogenized response. Finally, at the macro scale, the homogenized response is used to find the behavior of CNT-reinforced concrete under impact loading. The results indicate that less damaged areas are generated in the CNT-reinforced concrete model. It responses with higher resistance and energy absorption capacity, which results in considerable reduction of the penetration depth to contain a projectile.

1. Introduction

Nowadays, an increasing demand has generated for the production of high-performance engineering materials with outstanding mechanical properties for better protection of structures against extreme loadings such as impact and blast. The dynamic behavior of structures have further been brought to forefront of civil engineering research after catastrophes such as Oklahoma City bombing and the September 11. Hence, construction of structures and buildings with high performance materials against such phenomena is essential. Researchers try to probe deeper into designing high-strength structures to sustain extreme loadings. A reliable design of concrete structures depends significantly on the understanding of its dynamics properties. Therefore, realizing the response of concrete structures subjected to high strain rate explosive loadings becomes inevitable as the behavior of the concrete subjected to impact loading differs from the static loading due to the strain rate effects.

The major weakness of the cement is attributed to its weak strength and rapid crack growth in tension. Due to the brittle nature of the concrete, researchers have tried to improve its resistance and performance under explosive and impact loading through several methods such as addition of various types of fiber to the concrete (FRC). Several types of fiber can be implemented into the concrete such as steel, polypropylene and carbon fibers. They can increase ductility, fracture energy, mechanical strength and resistance against impact loadings and also prevent the crack propagation in the concrete.

Recently, several authors have studied the impact behavior of fiber-reinforced concretes. Among them are Sovjak et al. [1] who investigated the effects of impact loading on the response of Ultra-High Performance Fiber Reinforced Concrete (UHPFRC) slabs. Their results indicated that the optimal volume fraction of the fibers were 2% and using more than 2% did not improve the damage behavior. In addition, the dynamic impact behavior of the steel fiber reinforced concrete (SFRC) at different strain rates was studied by Xu et al. [2,3]. They illustrated that addition of fibers increased the stress and strain redistributions and the ductility of the sample. The SFRCs showed much enhanced crack-bridging effect at higher strain rates. Alavi Nia et al. [4] investigated the effects of different volume fractions of fibers on the impact behavior of fiber reinforced concrete (FRC). Their results indicated that fibers could increase the impact resistance of concrete specimen. They found that effect of fibers was more obvious in normal strength concrete than high-strength concrete and also steel fibers were more effective than polypropylene one. The studies of Nyström et al. [5] showed that the crack...
propagation and the penetration depth resistances of the fiber-reinforced concrete against projectile were improved and so they could be efficiently used in the protective structures.

A novel type of reinforcement for polymers, composites and also concrete is the carbon nanotube (CNT) [6], which is a very high strength material at the atomic scale. CNTs are hollow cylindrical structures made of carbon atoms at the nano scale. Its strength is more than a hundred time of steel and its elastic modulus is in the range of a Terapascal. Recently, several authors have studied the mechanical and engineering properties of the CNTs [7–9]. More details about these researches can be found in Ref. [10]. Inclusion of CNTs can improve the post-cracking behavior of the concrete material and consequently, CNT-reinforced concrete is expected to be a good candidate for blast-resistance structures by decreasing the penetration depth and damage of the structure.

In order to study the impact response of such material, it is essential to analyze the behavior of components of the concrete, and their collective response more accurately. Unfortunately, classical continuum theories cannot predict such complex behavior and more advanced methods should be adopted. Recently, several advanced computational methods such as meshfree [11–15] and multiscale methods were developed for fracture analysis [16,17] and crack propagation [18,19] in homogenous and heterogeneous materials. They estimated the material properties at the finer scales and used them as input parameters for the coarser one.

Moreover, several authors have studied the multiscale simulation of CNT-reinforced composites [20–22]. In a recent study, Eftekhari et al. [10] studied the quasi-static crack propagation of CNT-reinforced cements by the extended finite element method. Their results indicated that addition of CNTs to the cement paste, significantly increases the fracture energy and tensile strength of CNT-reinforced cement in static and quasi-static solutions. Furthermore, they illustrated that longer CNTs could increase the mechanical properties more than the shorter ones. In this research, the same methodology is further extended to investigate the effects of CNTs on the mechanical and impact behavior of the CNT-reinforced concrete using an advanced multiscale method. The physics of the model is extended to dynamic impact analysis and the effects of penetration of projectile and contact between the target and the projectile are considered.

Hence, at the atomic level, the mechanical properties of CNTs are investigated through the molecular dynamics simulation. Afterwards, at the micro scale, the mechanical properties of CNTs are incorporated in a finite element based cement hydration model to extract the mechanical properties of the CNT-reinforced cement. Then, at the meso scale, the concrete is modeled as a three-phase material, consisting of aggregates, cement paste reinforced by CNTs and interfacial transition zone (ITZ) which surrounds the aggregates. Finally, at the macro level, the effects of a projectile on the CNT-reinforced concrete specimen is investigated through the finite element simulation. A schematic representation of the mentioned procedure is depicted in Fig. 1.

2. Nano scale

First, the molecular structure of CNT is accurately simulated by the molecular dynamics (MD) approach. The LAMMPS (Large-scale atomic/molecular massively parallel simulator) open source code [23] is adopted to perform the MD simulations of CNTs under tensile and compressive loadings and to obtain the corresponding stress–strain curves.

In real applications, different types of CNTs may be created during the production and mixture processes. A chiral vector (n,m) defines the atomic configuration of the CNT structure. When n is equal to m, the CNT is called armchair and in the case of m equal to zero, it is called zigzag CNT. A CNT with a single tube is called single-walled CNT (SWCNT) whereas a multi-walled CNT (MWCNT) consists of several coaxial SWCNTs with an interlayer spacing of 0.34 nm. Two types of SWCNTs (diameter equal to 13.31 Å) and MWCNTs (diameter equal to 20.34 Å) with the length of 100 Å are considered in this study. SWCNTs and MWCNTs consist of 1650 and 4100 atoms, respectively.

To describe the interatomic interaction among the carbon atoms, the Tersoff potential function [24] is employed. In the Tersoff potential, the total potential energy of an atomic system is described as:

\[ E_{\text{total}}^{\text{nerg}} = \frac{1}{2} f_r (r_i) f_r (r_j) + b f_r (r_i) f_r (r_j) \]  

\[ f_r (r_i) = A_r \exp (-2 \alpha r_i) \]  

\[ f_A (r_i) = -B_r \exp (-2 \mu_i r_i) \]

\[ f_c (r_i) = \begin{cases} 1, & r_i \leq R_i \\ \frac{1}{2} + \frac{1}{2} \cos \left[ \pi \left( r_i - R_i \right) / (S_i - R_i) \right], & R_i < r_i < S_i \\ 0, & r_i \geq S_i \end{cases} \]

where \( f_r (r_i), f_A (r_i) \) and \( f_c (r_i) \) are the repulsive, attractive and cut-off functions, respectively, and \( r_i \) is the interatomic distance between
two carbon atoms \( i \) and \( j \). Moreover, \( A = 1.3936 \times 10^7 \text{ eV}, \quad B = 3.467 \times 10^5 \text{ eV}, \quad \lambda = 3.4879 \text{ Å}^{-1}, \quad \mu = 2.2119 \text{ Å}^{-1}, \quad \text{and} \quad R = 1.8 \text{ Å} \) and \( S = 2.1 \text{ Å} \). Further detail can be found in Ref. [24]. The Lennard–Jones (LJ) 6–12 potential is also adopted to describe the non-bonded interaction between carbon atoms at different layers of MWCNTs. The LJ effect is neglected for the interatomic distances more than 8.5 Å.

Axial displacement (both tension and compression) is applied at one side of the CNT with the constant rate of 0.01 Å/fs, while the atoms at the other side are constrained. The MD timestep is set to 1 fs and the simulation is performed for 1,000,000 time steps at the room temperature (300 K).

The average results of several simulations performed by Eftekhar at al [10], are summarized in Table 1 and Fig. 2. Table 1 presents the mean values of the model for tensile and compressive loadings. In addition, the average stress–strain curves for the CNT are depicted in Fig. 2. Clearly, the mechanical properties of CNTs are superior in tension than compression.

The failure/fracture mode shapes of CNTs in compression and tension are depicted in Fig. 3. Accordingly, in the tensile loading, all the section undergoes a uniform high stress and at the moment of failure, the stress of atoms which are located near the ruptured section, suddenly decreases. In contrast, in the compressive loading, the atoms which are located at the buckled section experience higher stresses than the other atoms.

### 3. Micro scale

The mean values of the results of nano scale simulations (Table 1) are used in the micro scale simulation of the CNT-reinforced cement. In this section, a particle kinetics chemical hydration model [25] is adopted to generate the finite element model for simulation of the CNT-reinforced cement. In the hydration model, only the contribution of the chemical reaction between tricalcium silicate and water is considered [26]. The results of the hydration simulation consists of three major portions of the hardened cement paste, including the unhydrated and hydrated phases and the capillary porosity. Smilauer and Bittnar [27] simulated three different sizes of the cement hydration model (25 μm, 50μm and 75 μm) and indicated that the size of the sample has negligible effect on the elastic modulus of hardened cement paste. Therefore, changing the RVE size at the micro scale does not have significant influence on the mechanical properties.

The dimension of the model is 50 × 50 μm with the thickness of 1 μm. The cross section of the carbon nanotubes is assumed as 0.05 μm². CNTs are dispersed in the cement with random length and orientation based on the model of Matsumoto and Nishimura [28] and a 2D truss element is adopted to model CNTs within the cement finite elements. A traction is applied at the boundary of the samples at the micro scale. The models are simulated by the open source finite element package OOFEM [29] under uniaxial loading in the plane stress condition. An isotropic damage behavior with linear softening regime is chosen for all components of the model:

\[
\sigma = (1 - \omega)E\varepsilon = f_i \left( 1 - \frac{\text{h ow/}f_i}{2G_f} \right)
\]

where \( \sigma \) and \( \varepsilon \) are the stress and strain, respectively, \( G_f \) is the fracture energy, \( h \) is the effective width of the finite element, \( f_i \) is the uniaxial tensile strength and \( \omega \) is the damage parameter. The mechanical properties of the cement paste phases (Table 2) are adopted from Ref. [30]. More information regarding this phase of simulation method can be found in Ref. [10]. A typical model of the mentioned procedure is depicted in Fig. 4a.

In order to verify the micromechanical simulation, the experimental results of Smilauer et al. [30] are used for verification. A cement paste model, which consists 3.47% volume fraction of 3 μm–CNT which correspond to the CEM86% + CHM14% experimental sample in Refs. [30], is investigated. The reference used a cement hybrid material (CHM) which CNTs were directly synthesized on the surface of cement grains. A 3-point bending test setup was used to calculate the mechanical properties of samples. The specimens were 13 × 13 ×80 mm in size with a 6 mm notch at the middle of the beam. The fracture energy of the experimental sample were in the range of 15–25 N/m (Fig. 4b). By comparing the fracture energy of the experimental tests with the prediction of micromechanical simulation (21.7 N/m), it is observed that the fracture energy of the numerical simulation is well within the range of experimental results.

Back to the micromechanical simulation to evaluate the mechanical properties of the CNT-reinforced cement, two models are considered: a plain paste cement without CNTs, and a CNT-reinforced cement with 3% volume fraction of CNTs with the length of 5 μm. Fig. 5 demonstrates the distribution of damage for tensile and compressive loadings. It is clear that adding CNT to the cement paste distributes the damaged area and prevents the stress concentration at a specific region. In addition, larger damaged area is observed in the compressive simulation than the tensile analysis.

The stress–strain curves for the models are depicted in Fig. 6. In both tension and compression cases, adding CNTs to the cement paste leads to a significant increase in the ultimate strength and fracture energy of the sample. In addition, the sample reinforced by CNTs can sustain more tensile and compressive strain than the cement; an indication of significant increase in ductility. The tensile strength of the cement is 2.28 Mpa at the strain level of 0.0002, but addition of CNTs can increase the tensile strength about 30% up to 2.96 Mpa at a higher strain of 0.0015. On the other hand, adding the CNT can increase the compressive strength of the cement from about 28.47 Mpa to 33.98 Mpa.

<table>
<thead>
<tr>
<th>Phase</th>
<th>( E ) (Gpa)</th>
<th>( \nu )</th>
<th>( \text{Fr} ) (Mpa)</th>
<th>( G_f ) (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unhydrated products</td>
<td>135</td>
<td>0.30</td>
<td>1800</td>
<td>118.5</td>
</tr>
<tr>
<td>Hydrated products</td>
<td>21.7</td>
<td>0.24</td>
<td>5.58</td>
<td>11.5</td>
</tr>
</tbody>
</table>

**Table 1**

Mechanical properties of CNTs.

**Table 2**

Mechanical properties of the cement paste phases [30].
4. Meso scale

Considering the complex composition of concrete, which contains a large number of aggregates connected through the cement paste, makes it difficult to create an accurate detailed model. At the meso scale, most of the researchers have assumed the concrete as a three phase material including, aggregates, interfacial transition zone (ITZ) around aggregates and the cement paste matrix [31]. Although it is possible to consider the porosity in this scale, it is ignored in this research due to the fact that its effects has already been considered at the micro level.

The material models for the cement paste and ITZ are assumed to be the isotropic damage model for the tensile and compressive failure with a linear softening regime [30], as stated in the micro level (Section 3). The cement properties are adopted from the results of micro scale simulations of Fig. 6. Different ITZ thicknesses have been adopted by several authors in their simulations [26,31,32].
However, Kim and Abu Al-Rub [32] proved that the thickness of ITZ did not significantly affect the behavior of concrete. Hence, a 200 μm thickness is chosen [32] to overcome the problems which may arise in the presence of very fine finite elements. ITZ is the weakest part of the concrete structure and the concrete behavior is mainly determined by the mechanical properties of this phase. With respect to the fact that the microstructure of ITZ is similar to a cement with higher percent of the water/cement ratio, the density of the cement paste in the vicinity of the aggregate's surface is lower than the other parts of the paste and the mechanical behavior of ITZ is assumed to be the same as the cement with values reduced to 70% of the cement [32].

Also, aggregates are assumed to be linear elastic with an elastic modulus of 70 Gpa, due to the fact that they usually remain undamaged during loading [33]. In addition, Pedersen et al. [34] mentioned that while the distribution of aggregates on mechanical properties of the concrete had negligible effects, the circular shape was the most common one used in the meso scale simulation of concrete. Different authors have used various volume fractions of aggregates. Among them, Qian [35] used 53% and Nguyen et al. [26] used 45% of the volume fraction of the aggregates. In this study, the volume fraction of aggregates is assumed 50%, which is a reasonable ratio for the randomly distributed aggregates.

For simulation of concrete, a 2D rectangular concrete sample with the dimension of 100 × 100mm is considered, which contains 3 phases of cement paste, ITZ and aggregates. In addition, only the large aggregates are simulated and the fine aggregates are assumed to be embedded within the cement paste [31]. Three types of aggregates with radii of 10 mm, 5 mm and 2.5 mm and the volume fraction of 5:3:2 are modeled. The mean element size is 3 mm. Two samples with the pure cement and CNT-reinforced cement are considered. The volume fraction and the length of CNTs are 3% and 5 μm, respectively. All simulations are performed in the plane stress condition. Triangular plane-stress elements are used. A traction is applied at the boundary of the samples at the meso scale. The left side of the models is constrained and a uniaxial displacement is incremented to overcome the problems which may rise to the fact that the microstructure of ITZ is similar to a cement with higher percent of the water/cement ratio, the density of the cement paste in the vicinity of the aggregate's surface is lower than the other parts of the paste and the mechanical behavior of ITZ is assumed to be the same as the cement with values reduced to 70% of the cement [32].

In order to evaluate the effects of the sample size dependency and mesh dependency at the meso scale, two extra concrete models without CNT (two 100mm × 100mm samples with mean element size of 2 mm and one 150mm × 150mm sample) are generated and analyzed under the compressive loading. The results of the damage distribution and stress–strain curves are depicted in Figs. 8 and 9, respectively. The results indicate that the variations of the sample size (sample size dependency) and element size (mesh dependency) have practically negligible effect on the overall behavior of the models.

The tensile damage and strain distributions in the models in the last step of the simulations are presented in Figs. 10 and 11, respectively. It is clearly observed that the damaged zones are mainly initiated in the ITZ area and then propagated to the cement paste. The aggregates are not affected and remain elastic. In addition, the crack pattern accumulates in a vertical direction. It can be concluded that there is a direct relationship between the tensile damage and strain and in the more damaged areas, the tensile strains are higher. The presence of CNTs in the cement paste generates the confinement effect and the crack bridging phenomenon in concrete. Consequently, the applied energy to the structure is stored in CNTs in the form of potential energy, which decreases the crack propagation rate and prevents the brittle failure of specimen.

The tensile stress–strain curves of the samples are illustrated in Fig. 12. In the case of concrete sample, the tensile strength is 1.84 Mpa, while for the CNT-reinforced concrete it is 2.43 Mpa; an increase of 32%. Furthermore, the ductility of the CNT-reinforced concrete is increased significantly with respect to the concrete such that the behavior the post-peak behavior becomes more stable and the area under the stress–strain curve increases significantly, about 6.6 times, which indicates the potential for more dissipation of the applied energy.

The compressive damage and strain distributions in the models in the last step of simulation are presented in Figs. 13 and 14, respectively. The overall responses remain similar to the tensile behavior. Comparing the damage distribution between the tensile and compressive loadings reveals that the damaged area in the compressive case propagates in the ITZ region and the cement paste zones in two planes, inclined 45° with respect to the horizontal axis.

The compressive stress–strain curve of the samples are depicted in Fig. 15. The compressive strength of the concrete is 18.1 Mpa, while this value for the CNT-reinforced concrete is 24.5 Mpa, which shows an increase of 26%. Furthermore, the ductility of the CNT-reinforced concrete is enhanced significantly with respect to the concrete (about 6.5 times). The results of the meso simulations are summarized in Table 3 and will be used in the macro scale impact simulations of Section 5.

5. Impact behavior at the macro scale

In order to study the mechanical properties of the CNT-reinforced concrete under impact loading, a typical impact problem is simulated by the finite element method. A concrete model [36], based on the combination of plasticity and shear damage models, which has successfully been implemented in several commercial finite element codes, is adopted to simulate the dynamic fracture behavior of a concrete specimen against a rigid projectile. In this material model, the stress state, is described based on the invariants of the stress tensor to determine the elastic limit surface (Yelastic) and the failure surface (Yfail). The initial elastic surface is derived from the failure surface by means of ratios of elastic tensile and compressive stresses to the corresponding ultimate strengths of concrete (f_{u0} (f) and (f_{u0} (f)) (Fig. 16).

The hardening surface (Y_{hard}) and the equivalent plastic hardening strain (\epsilon_{pl,hard}^{eq}) are described by Equations (5) and (6) based on the elastic and failure surfaces and by means of the equivalent plastic strain (\epsilon_{pl}^{eq}). In addition, the plastic stiffness is determined by the hardening slope G_{elastic}/G_{plastic} (36):

\[ Y_{hard} = Y_{elastic} + \frac{\epsilon_{pl}^{eq}}{\epsilon_{pl}^{eq,hard}} (Y_{fail} - Y_{elastic}) \]
\[ \varepsilon_{\text{eq}}^{\text{ult}} = \frac{(Y_{\text{ult}} - Y_{\text{elastic}})}{3G} \left( \frac{G_{\text{elastic}} - G_{\text{plastic}}}{G_{\text{elastic}}} \right) \]  

where \( G \) is the shear modulus. When concrete reaches its ultimate strength, the damage is controlled by the plastic strain and the effective strain associated with failure, which depends on the hydrostatic pressure \( p \). The effect of the strain rate plays a significant role on the response of the concrete structures under dynamic loading [37]. In the current concrete model, the strain rate effects are implemented through the increase of the fracture strength with the plastic strain rate. This model can cover both the tensile and compressive hardening and softening responses under different strain rates. Two different terms can be used for the compression and tension regimes in this model. The concrete material properties are

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**Fig. 8.** Two meso scale samples of concrete a) 100 mm × 100 mm, b) 150 mm × 150 mm.

**Fig. 9.** Effect of the sample size and mesh size on the mechanical properties.

**Fig. 10.** Damage distribution under tension; a) concrete, b) CNT-reinforced concrete.

**Fig. 11.** Tensile strain in X-direction; a) concrete, b) CNT-reinforced concrete.
listed in Table 4. Due to the fact that the focus of this research is on the concrete target and not the projectile, the projectile is assumed as a linear elastic steel object with the bulk and shear modulus of $2.05 \times 10^8$ kPa and $1.0 \times 10^8$ kPa, respectively. No erosion and failure can occur in the projectile. The material density of the projectile is 7.83 g/cm$^3$.

For the sake of simplicity, the model is simulated as an axisymmetric problem. The diameter and length of the projectile is 40 mm and 142 mm, respectively. In addition, the width and the height of the specimen is 300 mm and 250 mm, respectively. The target specimen is restrained from the top side. Due to the symmetry of the model, only half of the sample is modeled. The region which the projectile penetrates into the specimen is discretized with finer mesh than the other parts to increase the accuracy of the simulation, while keeping the total DOFs as low as possible (Fig. 17). The initial projectile impact velocity is set to 400 m/s. The Lagrange model is implemented to define the contact between the rigid projectile and the target.

To investigate the effect of mesh dependency on the response of the structure, three types of element sizes (0.25 mm, 0.5 mm and 1.0 mm corresponding to 30,000, 21,000, 15,000 elements, respectively) is considered for analysis of the CNT-reinforced concrete. The results of the velocity of projectile versus the penetration depth are depicted in Fig. 18. Except for the very coarse mesh, all fine meshes practically converge to a more or less similar result, proving the mesh-dependency of the final results.

It is clear that the velocity of the projectile decreases gradually during its path through the target. The projectile completely penetrates into the concrete target and exits from the other side of the model with a lower velocity. But in the case of the CNT-reinforced concrete target, the final velocity of the model becomes zero. In addition, at a similar level of penetration depth, the velocity of the projectile in CNT-reinforced concrete target is always smaller than the concrete one, which proves that CNT-reinforced concrete can better reduce the velocity of the projectile. The final speed of the projectile for the concrete and the CNT-reinforced concrete are 66 m/s
and 0 m/s, respectively. This means that the projectile should spend more energy for penetration and consequently, further reduction of speed.

**Fig. 19** presents the penetration depth of the model during the simulation time. In the case of the CNT-reinforced concrete, the projectile is completely stopped with a penetration depth of 209 mm in comparison to the concrete model with no ultimate penetration depth. The projectile passes through the concrete target with a finite velocity and continues its own way. It should be noted that, however, if the model was thick enough, the concrete target could finally stop the projectile but at a substantially larger distance compared with the CNT-reinforced concrete.

The damage contour of both models in the same time steps of the simulation are presented in **Fig. 20**. It is clear that in the CNT-reinforced concrete model, less damaged areas are generated at the final stages of the simulation as compared with the concrete model. This demonstrates that CNT-reinforced concrete has much resistance and high energy absorption capacity, which results in considerable reduction of the penetration depth. This is due to the fact that CNT reinforcement increases the fracture energy of the model and consequently the projectile should spend much more energy for penetrating into the target.

### 6. Conclusion

In this paper, the effects of carbon nanotubes on the mechanical and impact properties of CNT-reinforced concretes are investigated through the multiscale simulation. At the nano scale, CNTs are simulated by the molecular dynamics approach, with the important observation that the tensile properties of the CNT are superior than the compressive behavior. The results of the simulation at the meso scale indicate that addition of carbon nanotubes can increase the tensile and compressive strength about 32% and 26%, respectively. In addition, the ultimate speed and penetration depth of the projectile in the CNT-reinforced concrete are substantially decreased. Moreover, while the CNT-reinforced concrete stops the projectile completely, the projectile penetrates through and exits from the other side of the traditional concrete specimen with the finite velocity of 66 m/s.

<table>
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<th>Material properties</th>
<th>Concrete</th>
<th>CNT-reinforced concrete</th>
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<tr>
<td>Compressive strength (f_c) (Mpa)</td>
<td>18.1</td>
<td>24.5</td>
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<tr>
<td>Tensile strength (f_t) (Mpa)</td>
<td>1.84</td>
<td>2.43</td>
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<td>Shear modulus (G) (kpa)</td>
<td>792,083</td>
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<td>Bulk modulus (K) (kpa)</td>
<td>17,16,164</td>
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<td>G_elastic/G_plastic</td>
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<td>f_elastic/f_t</td>
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<tr>
<td>f_elastic</td>
<td>0.53</td>
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</tbody>
</table>

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**Fig. 16.** Elastic and failure surfaces for concrete [36].

**Fig. 17.** Discretization of the target model near the projectile.

**Fig. 18.** Effect of the mesh dependency on the penetration depth.

**Fig. 19.** Penetration depth of the projectile vs the time of the simulation.
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References


Fig. 20. Damage distribution in different times of the analysis.


