Corrections for

Extended Finite Element Method, Wiley/Blackwell, 2008

Chapter 2
Fracture Mechanics, A Review

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \end{Bmatrix} \tag{2.7}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{2.10}$$

$$\sigma_{xx} = \frac{\partial^2 \Phi}{\partial y^2} \tag{2.19}$$

$$\sigma_{yy} = \frac{\partial^2 \Phi}{\partial x^2} \tag{2.20}$$

$$\nabla^4 \Phi = \nabla^2 \left(\sigma_{xx} + \sigma_{yy} \right) = 0 \tag{2.22}$$

$$-\sigma_{\alpha\alpha} + \sigma_{\beta\beta} + 2i\sigma_{\alpha\beta} = 2\left[\bar{z}\psi''(z) + \chi''(z) \right] e^{2i\theta} \tag{2.32}$$

$$\begin{cases} \sigma_{rr} = 0 \\ \sigma_{\theta\theta} = \sigma_0 (1 + 2\cos 2\theta) \\ \sigma_{r\theta} = 0 \end{cases} \tag{2.42}$$

$$u_z = \begin{cases} 0 & \text{plane strain} \\ -\frac{\nu z}{E} (\sigma_{xx} + \sigma_{yy}) & \text{plane stress} \end{cases} \tag{2.72}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2 + \cos\frac{\theta}{2} \cos\frac{3\theta}{2} \right) \tag{2.73}$$

$$\sigma_{zz} = \begin{cases} \nu(\sigma_{xx} + \sigma_{yy}) & \text{plane strain} \\ 0 & \text{plane stress} \end{cases} \tag{2.76}$$

$$u_z = \begin{cases} 0 & \text{plane strain} \\ -\dfrac{\nu z}{E}(\sigma_{xx} + \sigma_{yy}) & \text{plane stress} \end{cases} \tag{2.80}$$

$$u_z = \frac{K_{\mathrm{III}}}{\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2} \tag{2.85}$$

$$\sigma_{r\theta} = \frac{K_{\mathrm{II}}}{\sqrt{2\pi r}}\left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right) \tag{2.91}$$

$$K_{\mathrm{I}} = \left[1.12 - 0.23\left(\frac{a}{2b}\right) + 10.56\left(\frac{a}{2b}\right)^2 - 21.74\left(\frac{a}{2b}\right)^3 + 30.42\left(\frac{a}{2b}\right)^4\right]\sigma_0\sqrt{\pi a} \tag{2.93}$$

$$\frac{\partial^2(\Pi + U_\Gamma)}{\partial a^2} \begin{cases} < 0 & \text{unstable fracture} \\ > 0 & \text{stable fracture} \\ = 0 & \text{neutral equilibrium} \end{cases} \tag{2.119}$$

$$K_I = \sigma\cos^2\theta_0\sqrt{\pi a} \tag{2.124}$$

$$K_{II}(\theta) = g(\theta)\left(K_{II}\cos\theta - \frac{1}{2}K_I\sin\theta\right) \tag{2.133}$$

$$\mathrm{G}(\theta) = \frac{1}{4\mathrm{E}'}\mathrm{g}^2(\theta)\left[\left(1 + 3\cos^2\theta\right)\mathrm{K}_{\mathrm{I}}^2 + 8\sin\theta\cos\theta\,\mathrm{K}_{\mathrm{I}}\mathrm{K}_{\mathrm{II}} + \left(9 - 5\cos^2\theta\right)\mathrm{K}_{\mathrm{II}}^2\right] \tag{2.136}$$

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Stress intensity factors can be determined at different radial distances from the crack tip by equating the numerically obtained displacements with their analytical expression in terms of the SIF. For plane **strain** problems in the *xy* plane (Fig. 2.10):

$$K_{\mathrm{I}} = \mu\sqrt{\frac{2\pi}{r}}\frac{u_y^b - u_y^a}{4(1-\nu)} \tag{2.147}$$

$$K_{\mathrm{II}} = \mu\sqrt{\frac{2\pi}{r}}\frac{u_x^b - u_x^a}{4(1-\nu)} \tag{2.148}$$

$$s = 2\sqrt{\frac{G_1 - G_2}{\kappa'}} \tag{2.163}$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}, \ \mathbf{J}^{-1} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi} \end{bmatrix} \tag{2.176}$$

$$\delta = r_p = \frac{1}{2\pi} \frac{K_I^2}{\sigma_{yld}^2} = \frac{a}{2} \left(\frac{\sigma_0}{\sigma_{yld}} \right)^2 \tag{2.189}$$

$$\sigma_n = \sigma_{xx} \cos^2 \theta + 2\sigma_{xy} \sin \theta \cos \theta + \sigma_{yy} \sin^2 \theta \tag{2.231}$$

$$\sigma_t = (\sigma_{yy} - \sigma_{xx}) \sin \theta \cos \theta + \sigma_{xy} (\cos^2 \theta - \sin^2 \theta) \tag{2.232}$$

$$u_t = -u_x \sin \theta + u_y \cos \theta \tag{2.234}$$

$$\begin{aligned} J = & \int w(-\sin\theta \mathrm{d}x + \cos\theta \mathrm{d}y) \\ & - \int [\sigma_{xx}\varepsilon_{xx} \cos\theta \mathrm{d}y + \sigma_{xy}\varepsilon_{yy} \sin\theta \mathrm{d}y - \sigma_{xy}\varepsilon_{xx} \cos\theta \mathrm{d}x - \sigma_{yy}\varepsilon_{yy} \sin\theta \mathrm{d}x \\ & + \sigma_{xy}\varepsilon_{xy} \cos\theta \mathrm{d}y - \sigma_{xy}\varepsilon_{xy} \sin\theta \mathrm{d}x + (\sigma_{xx} \sin\theta - \sigma_{xy} \cos\theta) \mathrm{d}u_n \\ & - (\sigma_{yy} \sin\theta + \sigma_{xy} \sin\theta) \mathrm{d}u_t] \end{aligned} \tag{2.235}$$

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W_\mathrm{s} \delta_{1j} \right] \frac{\partial q}{\partial x_j} \mathrm{d}A \tag{2.244}$$

$$J^{\mathrm{act}} = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W_\mathrm{s} \delta_{1j} \right] \frac{\partial q}{\partial x_j} \mathrm{d}A \tag{2.248}$$

$$J^{\mathrm{aux}} = \int_{A^*} \left[\sigma_{ij}^{\mathrm{aux}} \frac{\partial u_i^{\mathrm{aux}}}{\partial x_1} - W^{\mathrm{aux}} \delta_{1j} \right] \frac{\partial q}{\partial x_j} \mathrm{d}A \tag{2.249}$$

$$M = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_i^{\mathrm{aux}}}{\partial x_1} + \sigma_{ij}^{\mathrm{aux}} \frac{\partial u_i}{\partial x_1} - W^{\mathrm{M}} \delta_{1j} \right] \frac{\partial q}{\partial x_j} \mathrm{d}A \tag{2.250}$$

$$W_\mathrm{s} = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} \tag{2.251}$$

$$W^{\mathrm{aux}} = \frac{1}{2} \sigma_{ij}^{\mathrm{aux}} \varepsilon_{ij}^{\mathrm{aux}} \tag{2.252}$$

Chapter 3
Extended Finite Element Method for Isotropic Problems

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \qquad \mathbf{J}^{-1} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi} \end{bmatrix} \tag{3.7}$$

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W_s \delta_{1i} \right] \frac{\partial q}{\partial x_j} \, \mathrm{d}\Gamma \tag{3.14}$$

$$J = \sum_{\substack{elements \\ in\ A^*}} \left(\sum_{g=1}^{ng} \left\{ \left[\left(\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W_s \delta_{1i} \right) \frac{\partial q}{\partial x_j} \right] \det \left(\frac{\partial x_j}{\partial \xi_k} \right) \right\}_g W_g \right) \tag{3.15}$$

$$\mathbf{u}^h(\mathbf{x}_i) = \mathbf{u}_i + \sum_{k=1}^{m} p_k(\mathbf{x}_i) \mathbf{a}_{ik} \tag{3.34}$$

$$\delta(\xi) = \begin{cases} \frac{1}{2\beta} + \frac{1}{2\beta} \cos \frac{\pi \xi}{\beta} & -\beta < \xi < \beta \\ 0 & \text{otherwise} \end{cases} \tag{3.55}$$

$$\int_\Omega \boldsymbol{\sigma} \cdot \delta\boldsymbol{\varepsilon} \, \mathrm{d}\Omega = \int_\Omega \mathbf{f}^{\mathrm{b}} \cdot \delta\mathbf{u} \, \mathrm{d}\Omega + \int_{\Gamma_t} \mathbf{f}^{\mathrm{t}} \cdot \delta\mathbf{u} \, \mathrm{d}\Gamma \tag{3.79}$$

$$\mathbf{B}_i^{\mathbf{a}} = \begin{bmatrix} \left(N_i\left[H(\xi) - H(\xi_i)\right]\right)_{,x} & 0 \\ 0 & \left(N_i\left[H(\xi) - H(\xi_i)\right]\right)_{,y} \\ \left(N_i\left[H(\xi) - H(\xi_i)\right]\right)_{,y} & \left(N_i\left[H(\xi) - H(\xi_i)\right]\right)_{,x} \end{bmatrix} \tag{3.92}$$

$$H_{,i}(\xi) = \begin{cases} 1 & \text{at crack} \\ 0 & \text{otherwise} \end{cases} \tag{3.95}$$

Chapter 4
Static Fracture Analysis of Composites

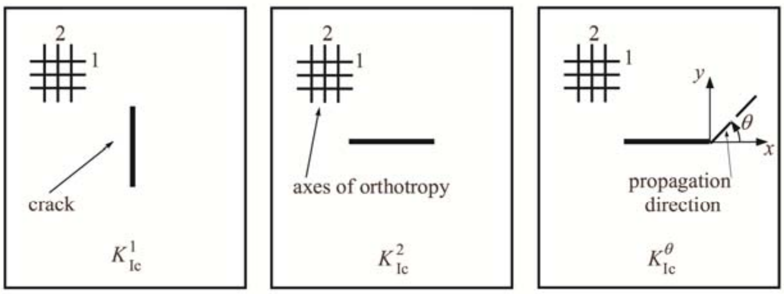

Figure 4.1 Fracture toughness for a homogeneous anisotropic solid.

$$\max\left\{\frac{1}{\cos^2\theta+\frac{K_{\mathrm{Ic}}^1}{K_{\mathrm{Ic}}^2}\sin^2\theta}\left[\mathrm{Re}\left[\frac{s_1t_1-s_2t_2}{s_1-s_2}\right]+\frac{K_{\mathrm{II}}}{K_{\mathrm{I}}}\mathrm{Re}\left[\frac{t_1-t_2}{s_1-s_2}\right]\right]\right\} \tag{4.27}$$

$$J=\int_\Gamma\left(W_{\mathrm{s}}\delta_{1j}-\sigma_{ij}\frac{\partial u_i}{\partial x_1}\right)\mathbf{n}_j\mathrm{d}\Gamma \tag{4.60}$$

$$J=\int_\Gamma\left(W_{\mathrm{s}}\delta_{1j}-\sigma_{ij}\frac{\partial u_i}{\partial x_1}\right)\mathbf{n}_j\mathrm{d}\Gamma \tag{4.121}$$

$$M^{(1)}=2t_{11}K_{\mathrm{I}}+t_{12}K_{\mathrm{II}} \tag{4.134}$$

$$\begin{cases}G_{\mathrm{I}}=-\dfrac{\pi}{2}K_{\mathrm{I}}c_{22}\,\mathrm{Im}\left[\dfrac{K_{\mathrm{I}}(s_1+s_2)+K_{\mathrm{II}}}{s_1s_2}\right]\\ G_{\mathrm{II}}=\dfrac{\pi}{2}K_{\mathrm{II}}c_{11}\,\mathrm{Im}\left[K_{\mathrm{II}}(s_1+s_2)+K_{\mathrm{I}}s_1s_2\right]\end{cases} \tag{4.29}$$

Chapter 5
XFEM for Cohesive Cracks

$$\mathbf{f}_{\mathrm{t}}^{\mathrm{c}}=\mathbf{f}_{\mathrm{t}}^{\mathrm{c}}\left(\delta\mathbf{u}_{\mathrm{n}},\delta\mathbf{u}_{\mathrm{t}}\right) \tag{5.20}$$

Figure 5.16 Equilibrium of a body with a cohesive crack.