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Contact Theory for Deformable Blocks in Three-Dimensional Discontinuous Deformation Analysis (3-D DDA)

Beyabanaki, S.A.R.

Pooyesh Rah Mandegar Consulting Engineers, No. 48, Shahr-Tash Alley, North Sohrevardi Avenue, Tehran 1559615311, Iran

Yeung, M.R.

Department of Civil Engineering, College of Engineering, California State Polytechnic University, 3801 W. Temple Avenue, Pomona, CA 91768, USA

Mohammadi, S.

School of Civil Engineering, University College of Engineering, University of Tehran, Tehran, Iran

Gao, Y. N.

State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, Jiangsu 221000, China

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ABSTRACT: In this study, contact theory (including contact detection and mechanics) of deformable blocks in nth-order Three-Dimensional Discontinuous Deformation Analysis (3-D DDA) is presented. When high-order 3-D DDA is employed, block faces may deform and not remain planar. In this case, conventional contact models cannot be used. To deal with this difficulty, the authors propose a simple technique. In this research, formulations of stiffness and force matrices in nth-order 3-D DDA due to normal and shear contact forces are presented, as well. One illustrative example is used to validate the new formulations and codes for high orders of displacement functions.

1. INTRODUCTION

The Discontinuous Deformation Analysis (DDA) method is a recently developed technique that is a member of the family of DEM methods. It is a pioneering method used to analyze the mechanical behavior of discrete blocks, In contrast, DDA as a complete block kinematics – a key component in dealing with interacting discrete blocks, guarantees the system equilibrium at any time, and is a real-time analysis. Both static and dynamic analyses can be conducted with the DDA method [1].

Original DDA formulation utilizes first order displacement functions to describe the block movement and deformation. Therefore, stress or strain is assumed constant through the block and the capability of block deformation is limited. This may yield unreasonable results when the block deformation is large and geometry of the block is irregular. In 2-D, to overcome these limitations, some approaches have been attempted. An approach to resolve this problem was to glue small blocks together to form a larger block using artificial joints [2] and sub-blocks [3]. Some researchers added finite element meshes in the blocks so that stress variations within the blocks can be accounted for [4-6]. An alternative approach is to include more polynomial terms in displacement function. Chern et al. [7] and Koo et al. [8] added the second-order to DDA. Ma et al. [9] and Koo and Chern [10] implemented the third-order displacement function in the 2-D DDA method. Hsiung [11] developed a more general formulation of the 2-D DDA.

There are some published papers on deformable blocks in 3-D DDA. Beyabanaki et al. [12-14] implemented Trilinear and Serendipity hexahedron FEM Meshes into 3-D DDA. Beyabanaki et al. [15-17] presented 3-D DDA with second- and third-order displacement functions. Recently, Beyabanaki et al. [18] presented 3-D DDA with n^{th}-order displacement functions, but they did not study its contact theory. In this paper, contact theory of nth-order 3-D DDA is presented. In this research, formulation of normal and shear contact forces are presented and applied to two examples.

2. APPROXIMATION OF GENERAL HIGH-ORDER DISPLACEMENT FUNCTIONS IN 3-D DDA

In 3-D DDA, the large displacements are an accumulation of the small displacements and deformations in a time step. Within each time step, the X, Y and Z displacements, (u, v, w), at any point (x, y, z) in a block can be represented using the approximation of a polynomial displacement function. In the original 3-D DDA, the block displacements function is equivalent to the complete first-order displacement approximation; constant strains and constant stresses are assumed within each block.

When displacement functions are taken as nth-order functions:

$$\begin{cases} u(x,y,z) = u_1 + u_2 x + u_3 y + u_4 z + u_5 xy + u_6 yz + u_7 xz + u_8 x^2 \\ \qquad + u_9 y^2 + u_{10} z^2 + \ldots + u_{p-2} x^n + u_{p-1} y^n + u_p z^n \\ v(x,y,z) = v_1 + v_2 x + v_3 y + v_4 z + v_5 xy + v_6 yz + v_7 xz + v_8 x^2 \\ \qquad + v_9 y^2 + v_{10} z^2 + \ldots + v_{p-2} x^n + v_{p-1} y^n + v_p z^n \\ w(x,y,z) = w_1 + w_2 x + w_3 y + w_4 z + w_5 xy + w_6 yz + w_7 xz + w_8 x^2 \\ \qquad + w_9 y^2 + w_{10} z^2 + \ldots + w_{p-1} x^n + w_{p-2} y^n + w_p z^n \end{cases} \tag{1}$$

which can be expressed as:

$$\begin{bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{bmatrix}_{3\times 1} = [C(x,y,z)]_{3\times 3p} \; \{D\}_{3p\times 1} \tag{2}$$

Then

$$[C(x,y,z)] = [[C_1] \quad [C_2]] \tag{3}$$

where

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & xy & 0 & 0 & yz & 0 & 0 & xz & 0 & 0 \\ 0 & 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & xy & 0 & 0 & yz & 0 & 0 & xz & 0 \\ 0 & 0 & 1 & 0 & 0 & x & 0 & 0 & y & 0 & 0 & z & 0 & 0 & xy & 0 & 0 & yz & 0 & 0 & xz \end{bmatrix}$$

$$[C_2] = \begin{bmatrix} x^2 & 0 & 0 & y^2 & 0 & 0 & z^2 & 0 & 0 & \ldots & x^n & \ldots & y^n & \ldots & z^n & 0 & 0 \\ 0 & x^2 & 0 & 0 & y^2 & 0 & 0 & z^2 & 0 & \ldots & x^n & \ldots & y^n & \ldots & z^n & 0 \\ 0 & 0 & x^2 & 0 & 0 & y^2 & 0 & 0 & z^2 & \ldots & x^n & \ldots & y^n & \ldots & z^n \end{bmatrix}$$

and the displacement variable vector $\{D\}$ is:

$$\{D\} = \{u_1 \; v_1 \; w_1 \; u_2 \; v_2 \; w_2 \; \ldots \; u_p \; v_p \; w_p\}^T \tag{4}$$

where $3p$ is the number of unknown coefficients:

$$p = \sum_{q=0}^{n} \frac{(q+1)(q+2)}{2} \tag{5}$$

The high-order function is necessary in most engineering analyses since it can represent stress concentrations within one block.

3. CONTACT THEORY

Contact analysis in DDA involves contact detection and contact mechanics. The former is mainly concerned with the geometric aspects of the method. The aim is to detect the overlap of discrete blocks and determine the contact normal and contact type. The latter is mainly concerned with the physical aspects of the method.

In the contact theory, it is necessary to determine the type of contact between any two blocks. The type of contact is important because it determines the mechanical response of the contact. There are many more types of contacts for 3-D blocks than for 2-D ones. In two dimensions, the contact types include corner-to-corner, corner-to-edge and edge-to-edge; whereas 3-D contact types include vertex-to-vertex, vertex-to-edge, vertex-to-face, edge-to-edge, edge-to-face and face-to-face. Beyabanaki et al. [19] presented a new contact calculating algorithm for contacts between two polyhedra in 3-D DDA. In this algorithm, all six type contacts in 3-D (vertex-to-face, vertex-to-edge, vertex-to-vertex, face-to-face, edge-to-edge, and edge-to-face) are simply transformed into the form of point-to-face contacts.

When a point-to-face contact candidate is found in the computation, the effect of the contact can be represented by applying two stiff contact springs in the normal and shear directions. The procedure of adding and removing stiff springs depending on the changes in contact states is known as ‘‘open-close’’ iteration. The global equation has to be solved repeatedly while selecting the lock or constraining positions.

When a high-order displacement function is employed, block faces may deform and not remain planar; therefore existing 3-D DDA contact detection schemes cannot be used directly. To deal with this difficulty, the authors propose a simple technique. In this method, a curved surface is divided into areas and may be approximated with flat polygons (Fig. 1).

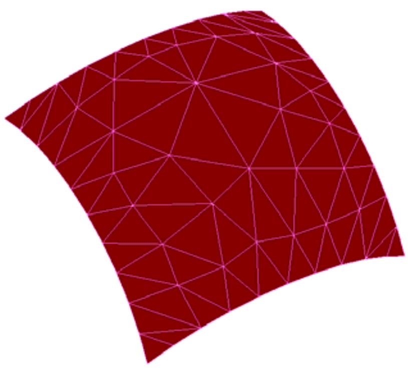

Fig. 1. Division of a curved face into flat polygons (sub-faces)

It is clear that the accuracy of current technique depends on the number of polygons. In fact, a curved face can be defined by some flat polygons named sub-faces here. Each sub-face can be considered as a plane in the original first-order 3-D DDA and its contact may be detected conventionally to calculate the contact sub-matrices when the time step is small.

As a result, because each block with curve faces are composed of some simplexes with flat faces, it is possible to do the volume integration by Shi's simplex integration method. Formulation of normal and shear contact submatrices are presented as follows.

3.1. Normal Contact Submatrices

As shown in Fig. 2, suppose P_1 is the vertex of a sub-face of face i before the displacement increment and P_1^* be the vertex after the displacement , $P_3P_4P_5$ is the contact sub-face of face j, (x_l, y_l, z_l) and (u_l, v_l, w_l) are the coordinates and small displacements of point P_l $(l = 0,...,5)$, respectively and $\vec{n}$ be the normal vector of the sub-face.

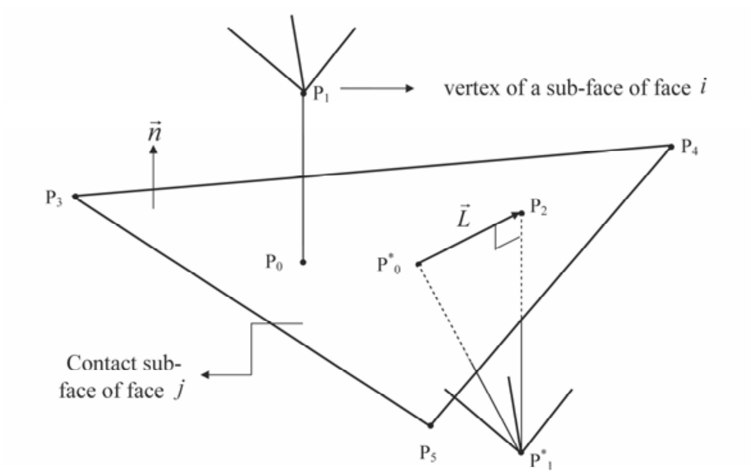

Fig. 2. Three-Dimensional contact (block i and a sub-face of block j)

If $P_3(x_3, y_3, z_3)$, $P_4(x_4, y_4, z_4)$ and $P_5(x_5, y_5, z_5)$ are the vertices of the sub-face, then:

$$\vec{n} = \overrightarrow{P_3P_5} \times \overrightarrow{P_3P_4} \tag{6}$$

The normal distance d_n from P_1^* to the sub-face is:

$$d_n = \frac{1}{l}\vec{n}\,.\,\overrightarrow{P_0^*P_1^*} = \frac{1}{l}\vec{n}\,.\begin{bmatrix}(x_1+u_1)-(x_0+u_0)\\(y_1+v_1)-(y_0+v_0)\\(z_1+w_1)-(z_0+w_0)\end{bmatrix} = \frac{1}{l}\vec{n}\,.\begin{bmatrix}x_1-x_0\\y_1-y_0\\z_1-z_0\end{bmatrix} + \frac{1}{l}\vec{n}\,.\begin{bmatrix}u_1-u_0\\v_1-v_0\\w_1-w_0\end{bmatrix} \tag{7}$$

The displacement of points P_1 and P_0 in Equation (7) can be substituted with the following forms:

$$\begin{Bmatrix}u_1\\v_1\\w_1\end{Bmatrix} = [C_i(x_1, y_1, z_1)].\{D_i\}$$

and

$$\begin{Bmatrix}u_0\\v_0\\w_0\end{Bmatrix} = [C_j(x_0, y_0, z_0)].\{D_j\} \tag{8}$$

Hence, Equation (7) can be written as:

$$d_n = \frac{1}{l}\vec{n}.\begin{bmatrix}x_1-x_0\\y_1-y_0\\z_1-z_0\end{bmatrix} + \frac{1}{l}\vec{n}([C_i][D_i]-[C_j][D_j]) \tag{9}$$

Let

$$M = \vec{n}.\begin{bmatrix}x_1-x_0\\y_1-y_0\\z_1-z_0\end{bmatrix} \quad , \quad [H] = \frac{1}{l}\vec{n}.[C_i(x_1, y_1, z_1)]$$

and

$$[Q] = \frac{1}{l}\vec{n}.[C_j(x_0, y_0, z_0)] \tag{10}$$

Therefore

$$d_n = \frac{M}{l} + [H][D_i] - [Q][D_j] \tag{11}$$

Using the penalty method, a mathematical spring with stiffness K_n is placed between point P_1 and the reference sub-face in the direction normal to the contact face. The potential energy of the normal spring is given by:

$$\Pi_{nc} = \frac{1}{2}K_n d_n^2 = \frac{1}{2}K_n\left(\frac{M}{l} + [H][D_i] - [Q][D_j]\right)^2 \tag{12}$$

By minimizing the potential energy Π_{nc}, the following $\sum_{l=0}^{n}\frac{3(l+1)(l+2)}{2} \times \sum_{l=0}^{n}\frac{3(l+1)(l+2)}{2}$ matrices can then be added to submatrices $[K_{ii}],[K_{ij}],[K_{ji}]$, and $[K_{jj}]$ in the global stiffness matrix:

$$[K_{ii}] = K_n [H]^T [H] \tag{13}$$

$$[K_{ij}] = K_n [H]^T [Q] \tag{14}$$

$$[K_{ji}] = K_n [Q]^T [H] \tag{15}$$

$$[K_{jj}] = K_n [Q]^T [Q] \tag{16}$$

And the following $\sum_{l=0}^{n} \frac{3(l+1)(l+2)}{2} \times 1$ vectors are calculated and then added to the global force vector:

$$[F_i] = -K_n \frac{M}{l} [H]^T \tag{17}$$

$$[F_i] = -K_n \frac{M}{l} [Q]^T \tag{18}$$

3.2. *Submatrix of Shear Contact*

As shown in Fig. 2, let P_2 be the projection of P_1 on the sub-face after the small displacement increment. Shear contact spring is applied to diminish the relative displacement of the two blocks when the shear force is smaller than the shear resistance of a discontinuity:

$$F_s < F_n tg(\varphi) + C \tag{19}$$

Where F_s is the shear contact force; φ is the friction angle of the discontinuity; and F_n is the normal contact force. Hence, the shear displacement along the $\overline{P_0^* P_2}$ is:

$$d_s = \sqrt{\left|\overrightarrow{P_0^* P_1^*}\right|^2 - d_n^2} \tag{20}$$

The potential energy of the shear spring is given by:

$$\Pi_{sc} = \frac{1}{2} K_s d_s^2 = \frac{1}{2} K_s \left(\left|\overrightarrow{P_0^* P_1^*}\right|^2 - d_n^2 \right)$$
$$= \frac{1}{2} K_s \left(\begin{bmatrix} (x_1+u_1)-(x_0+u_0) \\ (y_1+v_1)-(y_0+v_0) \\ (z_1+w_1)-(z_0+w_0) \end{bmatrix}^T \cdot \begin{bmatrix} (x_1+u_1)-(x_0+u_0) \\ (y_1+v_1)-(y_0+v_0) \\ (z_1+w_1)-(z_0+w_0) \end{bmatrix} - d_n^2 \right) \tag{21}$$

From Eq. (8) we have:

$$\Pi_{sc} = \frac{1}{2} K_s \left(\begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix}^T + [D_i]^T [C_i]^T - [D_j]^T [C_j]^T \right)$$
$$\times \left(\begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix} + [C_i][D_i] - [C_j][D_j] \right)$$
$$- \frac{1}{2} K_s \left(\frac{M}{l} + [H][D_i] - [Q][D_j] \right)^2 \tag{22}$$

By expanding and minimizing the potential energy Π_{sc}, the following matrices can then be added to the submatrices $[K_{ii}],[K_{ij}],[K_{ji}]$, and $[K_{jj}]$ in the global stiffness matrix:

$$[K_{ii}] = K_s [C_i]^T [C_i] - K_s [H]^T [H] \tag{23}$$

$$[K_{ij}] = -K_s [C_i]^T [C_j] - K_s [H]^T [Q] \tag{24}$$

$$[K_{ji}] = -K_s [C_j]^T [C_i] - K_s [Q]^T [H] \tag{25}$$

$$[K_{jj}] = K_s [C_j]^T [C_j] - K_s [Q]^T [Q] \tag{26}$$

And the $\sum_{l=0}^{n} \frac{3(l+1)(l+2)}{2} \times 1$ vectors $[F_i]$ and $[F_j]$ are calculated as follows and then added to the global force vector:

$$[F_i] = -K_s [C_i]^T \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix} + K_s \frac{M}{l} [H]^T \tag{27}$$

$$[F_i] = K_s [C_j]^T \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{bmatrix} + K_s \frac{M}{l} [Q]^T \tag{28}$$

4. ILLUSTRATIVE EXAMPLE

The formulation described in the previous sections has been programmed in VC++. To investigate it, an illustrative example is presented.

As shown in Fig. 3a, this case involves a block falling. In this example, left side of block *B* is fixed, and block *A* falls down due to the gravity force. Blocks *A* and *B* are $1.5m \times 1.5m \times 1.5m$ and $8m \times 1m \times 3m$, respectively. The values for the Poisson's ratio and mass density for each block are 0.2, and 3600 kg/m3, respectively. Block *A* falls freely initially and after impacting, Block *B* bends. The deformation of the block system is shown in Fig 3a-c. It indicates that using the sub-face technique, contact modeling is easily possible when block faces are not planar.

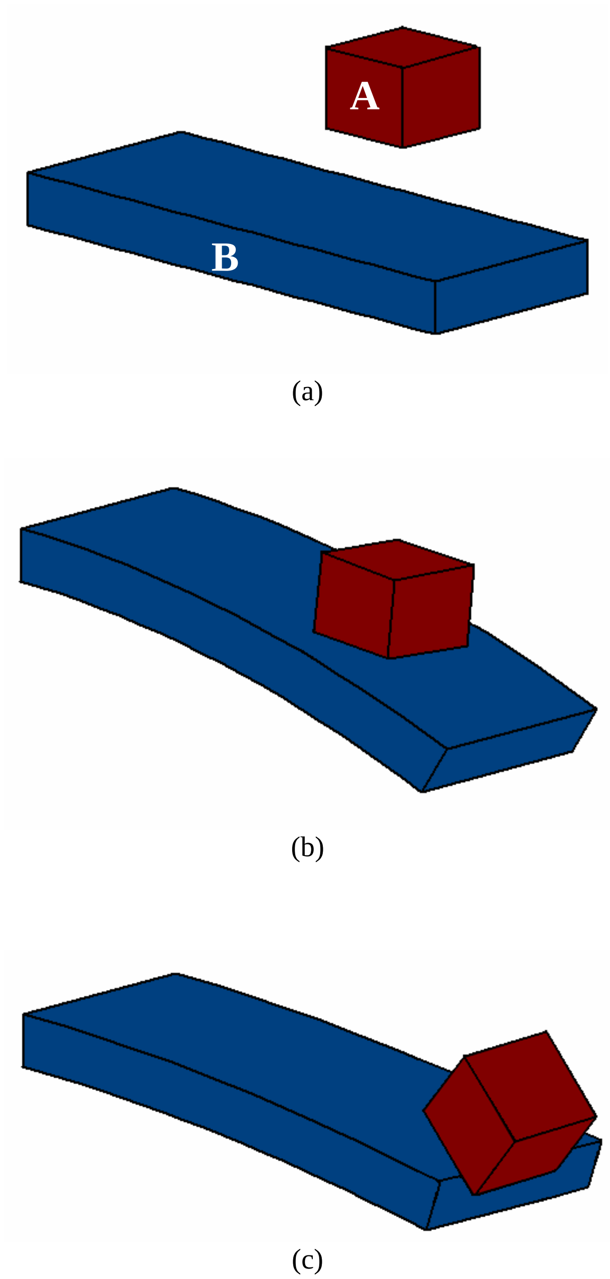

Fig. 3. a) Initial configuration of the block system, b) the deformation of the blocks after 950 steps, c) the deformation of blocks after 1350 steps.

5. CONCLUSIONS

Recently, 3-D DDA with high-order displacement functions is presented, but its contact theory is not studied. In this paper, contact theory for deformable blocks in high-order 3-D DDA and formulation of normal and shear contact forces are presented. When 3-D DDA with high-order displacement functions are employed, block faces may deform and not remain planar. To solve this problem, sub-face technique is proposed; in this method, a curved surface is divided into some flat areas. The presented example shows that the derived formulations of stiffness and force matrices in High-order 3-D DDA due to normal and shear contact forces and the provided code work very well.

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