DISCONTINUUM APPROACH FOR DAMAGE ANALYSIS OF COMPOSITES

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1 INTRODUCTION

It is well known that laminated composites are vulnerable to transverse impact loadings, which might cause significant internal damage. In general, according to the orthotropic laminated nature of composites, the failure modes may be classified into four different types : matrix failure, delamination, shear cracking, and erosion damage. The three flexural modes are not always activated in an impact loading of composites. If the projectile has sufficient energy, it may penetrate into the laminate and may cause a large front surface erosion without activating other behaviours [1].

It has also been observed that delamination and inplane fracture in composites subjected to low or high velocity impact loadings, are progressive phenomena which may rapidly propagate throughout the component. This might result in the creation of new totally separated parts, which interact with their surrounding regions. Consequently, a powerful scheme is required to be able to monitor the fracturing process and to effectively model both individual and interaction behaviour.

The *discrete element method* idealizes the whole medium into an assemblage of individual bodies, which in addition to their own deformable response, interact with each other to perform the same response as the medium. This discontinuum approach may provide a better algorithm for dealing with highly fractured regions which the finite element method is not suited to, since it is rooted in the concepts of continuum mechanics and necessitates that discontinuities be propagated along the predefined element boundaries. A far more general approach then, might be offered by a combination of both methods [2].

2 COMBINED FINITE/DISCRETE ELEMENT METHOD

Figure 1 shows a typical combined finite/discrete element mesh for a quarter of a composite plate subjected to concentrated central loading. In a combined FE/DE method, the possible fractured region is modelled using a discrete element mesh and the rest of the specimen is modelled by a standard finite element mesh. A combined mesh enables us to prevent unnecessary contact detection and interaction calculations which comprise a major part of the analysis time.

Each group of similar layers is modelled by one discrete element. Each discrete element is discretized by a finite element mesh and might have material or geometric nonlinearities. The interlaminar behaviour of discrete elements is governed by bonding laws, including contact and friction interactions for the post delamination phase. Interactions between finite elements and discrete elements are modelled by transition interfaces (Figure 1). All interfaces are controlled against the delamination criterion. Once two layers are delaminated, the corresponding interface will still be capable of further contact and friction interaction.

Inplane fracture may result in the creation of new discrete bodies which are in contact and friction interaction with neighbouring bodies (Figure 1b). A special remeshing algorithm is adopted to maintain compatibility conditions in newly fractured regions.

An *alternating digital tree* [3, 4] contact detection algorithm is employed to detect the possibility of contact between discrete elements. Contact forces, have to be evaluated to define the subsequent motion of the discrete elements from the dynamic equilibrium equation.

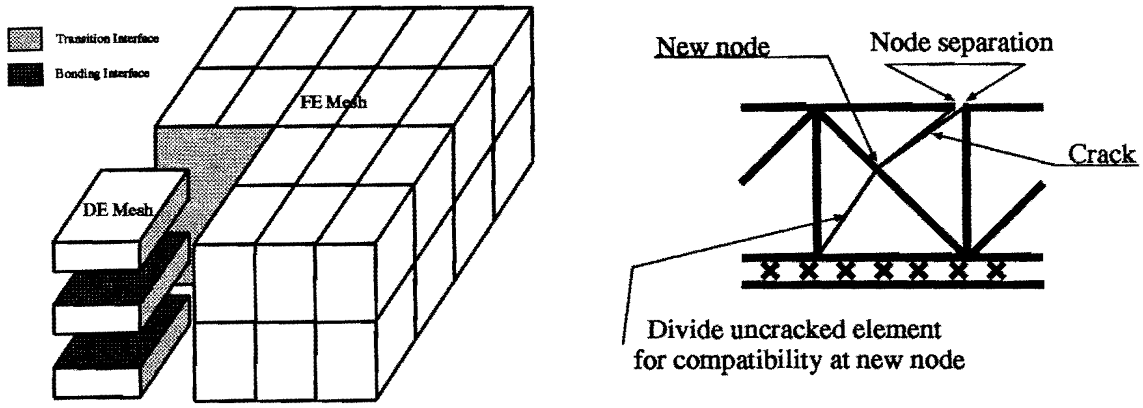


Figure 1: Discrete element modelling; a)Combined FE/DE mesh, b)Fracturing process.

2.1 Contact interaction

The spatial version of the weak form of the dynamic boundary value problem for any admissible displacement variation $\delta\mathbf{u}$ may be defined as

$$\int_{\Omega} \delta\boldsymbol{\epsilon}(\boldsymbol{u}) : \boldsymbol{\sigma}(\boldsymbol{u}) dv + \int_{\Omega} \delta\boldsymbol{u} \cdot \rho\ddot{\boldsymbol{u}} dv = \int_{\Omega} \delta\boldsymbol{u} \cdot \boldsymbol{f}^{\text{body}} dv + \int_{\Gamma_\sigma} \delta\boldsymbol{u} \cdot \boldsymbol{f}^{\text{surf}} da + \int_{\Gamma_c} \delta\mathrm{g}(\boldsymbol{u}) \cdot \boldsymbol{f}^{\text{con}} da \quad (1)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\epsilon}$ is the strain tensor, $\boldsymbol{u}$ is the displacement vector, while g represents the contact gap vector corresponding to a penalty formulation of contact interaction. Standard finite element discretisation of the variational form (1) results in the discrete set of algebraic time dependent equations which may be expressed, in matrix form, as

$$\boldsymbol{f}^{\text{int}}(\boldsymbol{u}, t) + \boldsymbol{M}\ddot{\boldsymbol{u}}(t) = \boldsymbol{f}^{\text{ext}}(t) + \boldsymbol{f}^{\text{con}}(t) \quad (2)$$

A central difference method with variable time steps is then employed, with a lumped mass matrix, to solve (2) for displacement, velocity and acceleration in a time incrementation procedure.

The component form of the virtual work of the contact forces associated with the contact node may be evaluated by [5]:

$$\delta\mathcal{W}^{\text{con}}(\delta\boldsymbol{u}) = f_k^{\text{con}}\, \delta\mathrm{g}_k = f_k^{\text{con}} \frac{\partial \mathrm{g}_k}{\partial u_i^s}\, \delta u_i^s \quad (3)$$

where $k = n, t$ and u_i^s is the i-component of the displacement vector at node s, $\mathrm{g} = (\mathrm{g}_n, \mathrm{g}_t)$ is the relative motion (gap) vector, and $\boldsymbol{f}^{\text{con}}$ is the contact force vector over the contact area A^c,

$$\boldsymbol{f}^{\text{con}} = A^c\, \boldsymbol{\sigma}^c \quad , \quad \boldsymbol{\sigma}^c = \boldsymbol{\alpha}\, \mathrm{g} \quad (4)$$

Here $\boldsymbol{\alpha}$ is the penalty term matrix, which can vary for normal and tangential gaps and even between single contact nodes.

2.2 Crack initiation criteria

The Chang-Springer criterion may be properly used for predicting the initiation of delamination [6]:

$$(\frac{{\sigma_z}^2}{N^2}) + (\frac{{\sigma_{xz}}^2 + {\sigma_{yz}}^2}{T^2}) = d^2 \quad , \quad \begin{cases} d < 1 & \text{no failure} \\ d \geq 1 & \text{failure} \end{cases} \quad (5)$$

where N and T are the unidirectional normal and tangential strengths of the bonding material, respectively. The imminence of inplane failure is monitored by the orthotropic Tsai-Wu criterion [7]:

$$\begin{aligned}\Phi(\boldsymbol{\sigma}) \;=\; & \alpha_{23}(\sigma_2-\sigma_3)^2+\alpha_{31}(\sigma_3-\sigma_1)^2+\alpha_{12}(\sigma_1-\sigma_2)^2+\alpha_{11}\sigma_1+\alpha_{22}\sigma_2+\alpha_{33}\sigma_3+ \\ & 3\alpha_{44}{\sigma_{23}}^2+3\alpha_{55}{\sigma_{31}}^2+3\alpha_{66}{\sigma_{12}}^2-\bar{\sigma}^2(\kappa)\end{aligned} \qquad (6)$$

where the α coefficients are evaluated based on twelve tensile and compressive normal and shear ply strengths. Lack of accurate material parameters usually leads to usage of simplified versions of this criterion.

A bilinear local softening model is adopted in this study to account for release of energy and redistribution of forces which caused the formation of a crack [4].

3 NUMERICAL RESULTS

Two tests are considered. Firstly, a multilayer plate subjected to transverse impact is analysed to investigate the pure delamination failure. In the second test case, a composite bend is simulated to demonstrate the capability of the method for modelling progressive inplane fracture and delamination in composites.

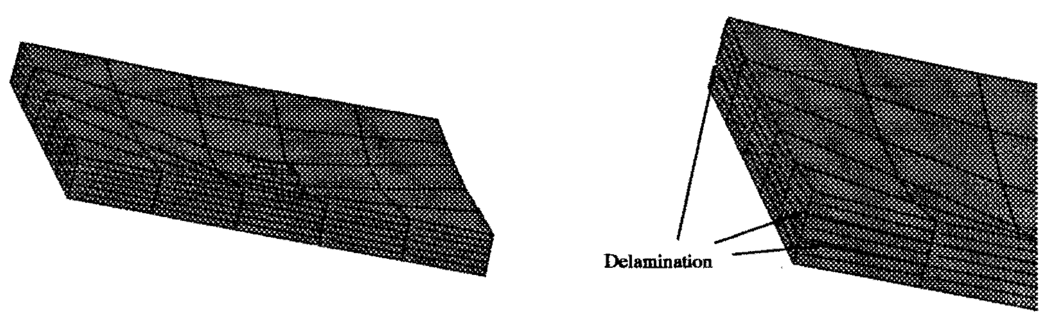
3.1 Composite plate

A layered plate subjected to a central impact loading is considered. The behaviour of the plate is investigated for different adhesive strengths between the layers. Figure 2 represents the delamination patterns for high and normal strength bondings. The free edge delamination commenced from the corner nodes of the plate and then propagated toward the centre of the plate.

3.2 Composite bend

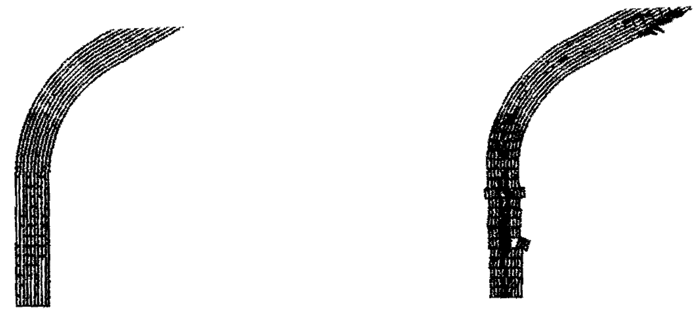
A 120-degree $[0_n, 90_n, 0_n]$ composite bend subjected to downward concentrated loading on its top end is considered. Each laminate is composed of Fiberite T300/1034-C graphite epoxy unidirectional tape [6].

An eight layer discrete element model was implemented for modelling the bend. Apart from early local fractures in the vicinity of the applied load, the progressive fracturing commenced near to the clamped edge of the bend and concentrated in the weak mid-layer of the bend (Figure 3a). It then spread across the thickness of the bend up to final collapse (Figure 3b).



(a) Bonded layers. (b) Free edge delamination.

Figure 2: Delamination analysis of laminated plate.



(a) Early fractures. (b) Threshold of collapse

Figure 3: Deformed shape and fracture layouts of 120-degree bend subjected to inward loading.

4 CONCLUSIONS

A combined finite/discrete element method has been successfully developed for damage analysis of composites. The interlaminar behaviour and the interaction between the fractured regions, predicted by the Chang-Springer and a simplified Tsai-Wu criterion respectively, are modelled using a penalty based contact algorithm. Several tests have been used to assess the performance of the method.

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