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Fracture analysis of FRP composites using a meshless finite point collocation method

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ABSTRACT: With the wide application of FRP composites for retrofitting and strengthening of existing structures, accurate and efficient analyses of models become very important as the study of interface crack growth is essential to ensure the reliability of structures under static and cyclic loading conditions. In the absence of exact analytical solutions for complex plate problems, numerical methods such as the finite element method, the boundary element method, and recently meshless methods have been frequently used and extended to include nonlinear effects. In the past decade, meshless methods have been widely developed and implemented to various engineering problems. Krysl and Belytschko have extended the element free Galerkin (EFG) method for static analysis of thin plates and shells. All EFG formulations require a background mesh of cells for integration. In a different meshless approach, the proposed truly meshless Finite Point Method (FPM) uses a moving least square approximation within a collocation strong form for solving the governing equilibrium equation. In FPM, imposition of boundary conditions is not directly performed. Instead, a procedure similar to other internal nodes is followed. In this paper, the fracture analysis of two dimensional FRP composites is considered. Enriched basis functions are introduced in the meshless formulation to capture the singularity at potential crack tips. A number of problems are solved and the results are compared with available analytical solutions and other numerical techniques to assess the performance of the proposed approach.

1 INTRODUCTION

The needs of modeling orthotropic materials have been recently revived with great interest, having enormous applications in various structural systems like aerospace and automobile industries. The main advantages of using these materials can be attributed to their high stiffness and low ratio of weight to strength. Some analytical investigations have been reported on the fracture behaviour of such materials such as the pioneering one by (Muskelishvili 1952) and (Sih & Paris & Irwin 1965 ; Tupholme 1974 ; Ting 1996 ; Asadpoure & Mohammadi 2007). Numerical methods, however, have been widely utilized for solving different mechanical problems. Despite the fact that the finite element method is more convenient and applicable because of its ability in modeling complex behaviors, meshless methods have been increasingly adapted for simulation of complex problems such as crack initiation and propagation problems. For elasticity problems(Oñate & Perazzo & Miquel 2001) presented a finite point method using the FIC technique in order to overcome the instability of the results obtained from the conventional version of FPM. (Boroomand & Tabatabaei & Oñate 2005) proposed a mapping scheme to reduce the problem of ill-conditioning of the coefficient matrix which sometimes occurs due to non-isotropic arrangement of the points. This stabilization procedure dramatically improves the convergence and accuracy of the method. (Bitaraf & Mohammadi 2006) solving the chloride diffusion equation for prediction of service life of concrete structures and initiation time of corrosion of reinforcements.

In this study, the truly meshless finite point method (FPM) is combined with principles of partition of unity and the extended finite element method to accurately simulate singular fields around a crack tip and the discontinuous field along the crack. FPM is a strong form solution and can be regarded as a continuous extension of the finite difference method based on the moving least square approximation.

2 FRACTURE MECHANICS FOR 2D ORTHOTROPIC MATERIALS

The stress-strain law in an arbitrary linear elastic material can be written as

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl} \quad (i, j, k, l = 1,2,3) \tag{1}$$

where ε_{ij} and σ_{kl} are linear strain and stress tensors, respectively, and s_{ijkl} is a fourth-order compliance tensor. By introducing the contracted form (Lekhnitskii 1963), the stress-strain relation for a plane stress problem can be re-written as

$$\varepsilon_{\alpha} = a_{\alpha\beta}\sigma_{\beta} \qquad (\alpha, \beta = 1,2,6) \tag{2}$$

Now assume an anisotropic body is subjected to arbitrary forces with general boundary conditions and a crack. Global Cartesian co-ordinate (X_1, X_2), local Cartesian co-ordinate (x, y) and local Polar co-ordinate (r, θ) defined on the crack-tip are illustrated in Figure 1. A fourth-order partial differential equation with the following characteristic equation can be obtained using equilibrium and compatibility conditions(Lekhnitskii 1963)

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0 \tag{3}$$

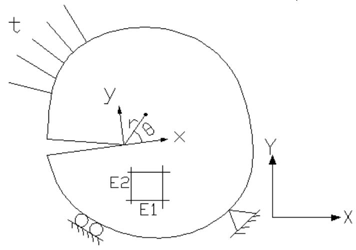


Figure 1. An arbitrary orthotropic cracked body subjected to traction **t**.

The two dimensional displacement field in the vicinity of the crack-tip have been previously derived by (Sih & Paris & Irwin 1965) by means of analytical functions and complex variables, $z_k = x + \mu_k y_k$, $(k = 1,2)$. The displacement components for pure mode I are:

$$u_1 = K_I\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left(\frac{1}{\mu_1 - \mu_2}\left\{\mu_1 p_2\sqrt{\cos\theta + \mu_2\sin\theta} - \mu_2 p_1\sqrt{\cos\theta + \mu_1\sin\theta}\right\}\right) \tag{4}$$

$$u_2 = K_I\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left(\frac{1}{\mu_1 - \mu_2}\left\{\mu_1 q_2\sqrt{\cos\theta + \mu_2\sin\theta} - \mu_2 q_1\sqrt{\cos\theta + \mu_1\sin\theta}\right\}\right) \tag{5}$$

where p_k and q_k are defined as:

$$p_k = a_{11}\mu_k^2 + a_{12} - a_{16}\mu_k \ , \ q_k = a_{12}\mu_k + \frac{a_{22}}{\mu_k} - a_{26} \tag{6}$$

(Lekhnitskii 1963) showed that the roots of (Eq. 3) are always complex or purely imaginary ($\mu_k = \mu_{kx} + i\mu_{ky}$, $(k = 1,2)$) and occur in conjugate pairs as μ_1, $\bar{\mu}_1$ and μ_2, $\bar{\mu}_2$.

3 ENRICHED FINITE POINT METHOD

In this section an overview is given to the approximation used to construct the shape functions of the proposed FPM.

3.1 *Weighedt least square method*

Least square methods are among the efficient procedures frequently used for approximation of unknown functions in meshless methods. Assume that a function *u(x)* is to be approximated in a domain Ω with *n* nodal points. The approximate function in sub-domain Ω_i in the vicinity of the *i*th node with n_i neighboring nodes may be written as

$$u(x) \cong \hat{u}(x) = \sum_{i=1}^{m} p_i(x)\alpha_i \qquad (7)$$

where p(x) is a vector of base monomials and α is a vector of coefficients. Assuming that generally n_i is greater than m, then satisfaction of (7) requires a least square procedure, which leads to minimization of the following discrete norm:

$$J = \sum_{j=1}^{n_i} w_i(x_j)[\bar{u}_{j-} \hat{u}(x_j)]^2 = \sum_{j=1}^{n_i} w_i(x_j)[\bar{u}_{j-} p^T(x_j)\alpha]^2 \qquad (8)$$

where w_i (x) is a suitable weighting function for the ith sub-domain. Note that for each domain a local co-ordinate system is defined; the origin of which is located at the master node of the domain (node number i). The discrete norm in Equation (8) is minimized as

$$\frac{\partial J}{\partial \alpha} = 0 \qquad (9)$$

which yields to the following system of equations:

$$A\alpha = B\bar{u} \qquad (10)$$

where

$$A = \sum_{j=1}^{n_i} w_i(x_j)p(x_j)p^T(x_j) \qquad (11)$$

and

$$B = [w_i(x_1)p(x_1),..., w_i(x_{n_i})p(x_{n_i}) \qquad (12)$$

Solution of (10) gives

$$\alpha = A^{-1}B\bar{u} \qquad (13)$$

The approximation for the ith sub-domain is obtained by substituting Equation (13) into (7)

$$\hat{u}(x) = p^T(x)A^{-1}B\bar{u} = N(x)\bar{u} \qquad (14)$$

where N(x) is a matrix containing the shape functions for each domain.

3.2 *Crack tip enrichment*

One way to enrich the FPM formulation for modeling singular stress fields around a crack tip is to include the leading terms of the near-tip asymptotic expansion for the displacement field in the basis function. In the full intrinsic enrichment of FPM approximation for orthotropic fracture problems, the entire near-tip asymptotic displacement terms are added to the linear terms of the basis:

$$p^T(x) = \begin{bmatrix} 1, x, y, \sqrt{r}\cos\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\cos\frac{\theta_2}{2}\sqrt{g_2(\theta)}, \\ \sqrt{r}\sin\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\sin\frac{\theta_2}{2}\sqrt{g_2(\theta)} \end{bmatrix} \qquad (15)$$

where

$$g_k(\theta) = \sqrt{(\cos\theta + \mu_{kx}\sin\theta)^2 + (\mu_{ky}\sin\theta)^2} \qquad ,(k=1,2) \qquad (16)$$

$$\theta_k = arctg\left(\frac{\mu_{ky}\sin\theta}{\cos\theta + \mu_{kx}\sin\theta}\right) \qquad ,(k=1,2) \tag{17}$$

4 NUMERICAL SIMULATION

4.1 *Crack analysis in an isotropic plate*

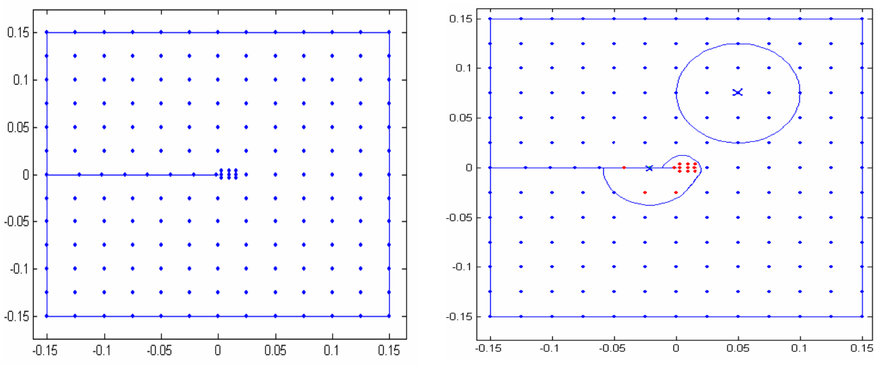
A closed form solution for a crack can be constructed by using the well-known near- tip field in a domain around the crack tip and prescribing the displacement along the boundaries accordingly. This can be considered as a patch test for singular fields. A square patch with side length of *2d* and a crack length *d* is considered. The exact solutions for the stress field are given by:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos(\frac{\theta}{2})\left[1+\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})\right], \sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos(\frac{\theta}{2})\left[1-\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})\right] \tag{18}$$

Also, the quartic spherical weight function with a radius of d_{ml} is adopted:

$$w(x_I,x) = w(r) = \begin{cases}(1-6r^2+8r^3-3r^4) & for \quad 1>r>=0, \\ 0 & for \quad r>=1.\end{cases}, r = \|x_I - x\|/d_{ml} \tag{19}$$

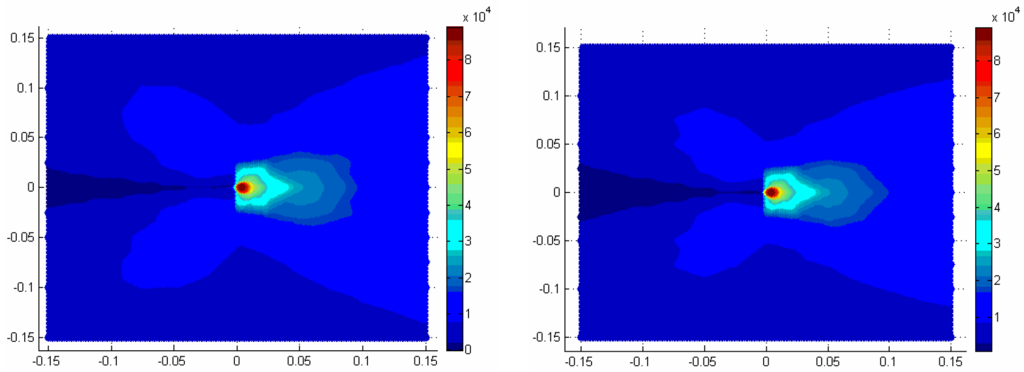
Figure 2 illustrates the nodal distribution and selection of support domains for a typical node and a crack tip. The total number of nodes is 187, and each cloud includes about 12 nodes.



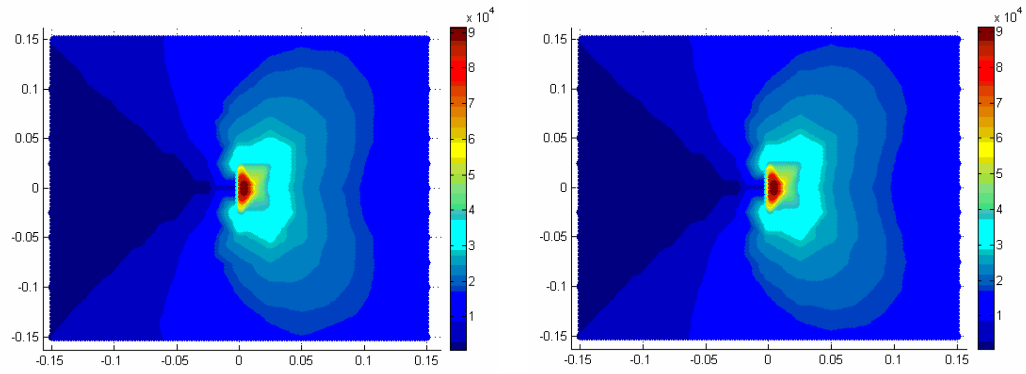
(a) (b)

Figure 2. a) nodal distribution, b) support domain of node x_{kp}

Figure 3 depicts the mode I stress contours over the cracked plate obtained by the proposed approach in comparison with the exact solution (Eq.18). The low level of generated error shows the quality of the numerical predictions by the proposed enriched finite point method.



(a) (b)



(c) (d)

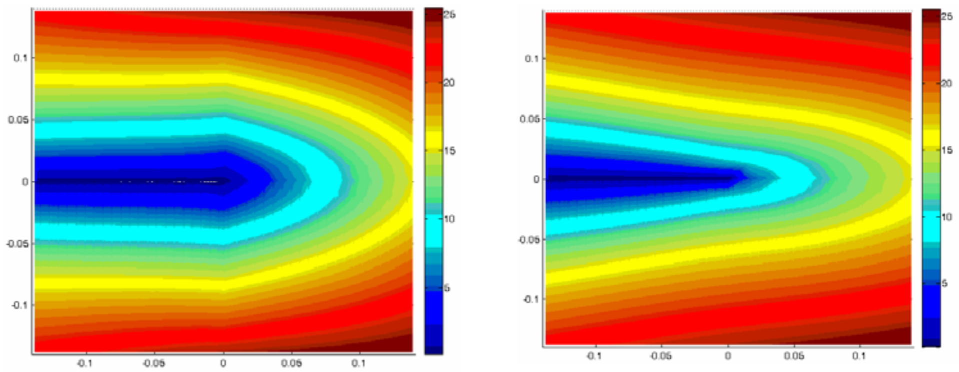
Figure 3. Stress contours: a) σ_{xx} (FPM), b) σ_{xx} (exact), c) σ_{yy} (FPM), d) σ_{yy} (exact).

4.2 Orthotropic FRP plate

A square patch test with a side length 2d and a crack length d is considered. The plate is composed of a graphite-epoxy material with the following orthotropic properties:

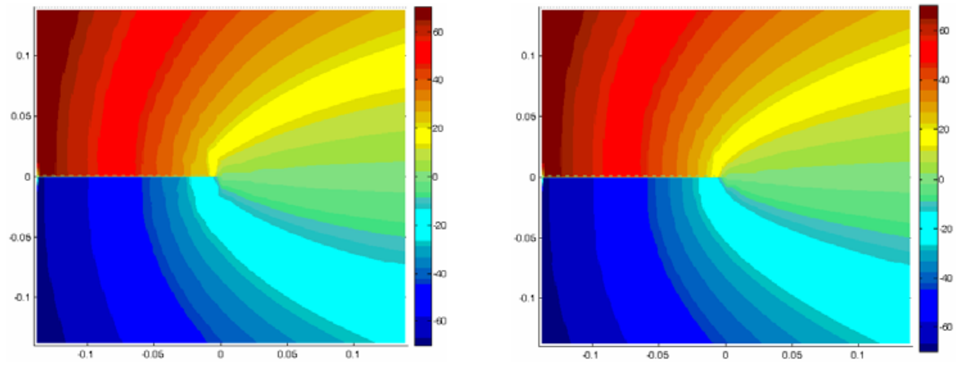
E_1 = 114.8 GPa, E_2 = 11.7 GPa, G_{12} = 9.66 GPa, υ_{12} = 0.21

Figure 4 and figure 5 depicts the mode I displacement contours over the cracked plate obtained by the proposed approach in comparison with the exact solution (Eq.4 and Eq.5). Similar to isotropic plate, the total number of nodes is 187 and each cloud includes at least 12 nodes.



(a) (b)

Figure 4. Displacements contours: a) u_x (FPM), b) u_x (exact).



(a) (b)

Figure 5. Displacements contours: a) u_y (FPM), b) u_y (exact).

5 CONCLUSIONS

A truly meshless enriched finite point approach has been presented for solving the governing equations of FRP cracked plates. FPM is a strong form solution based on the moving least square approximation for the displacement field variable. Enriched basis functions have been introduced in the meshless approximation to capture the singularity of the stress field at crack tips. Numerical results have shown good agreement with available analytical solutions.

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