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Modeling delamination in composite laminates using XFEM by new orthotropic enrichment functions

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Abstract. In this research, the extended finite element method (XFEM) is improved for modeling interfacial cracks between two orthotropic media by new set of orthotropic enrichment functions. New set of bimaterial orthotropic enrichment functions are developed and utilized in XFEM analysis of linear elastic fracture mechanics of layered composites. Interlaminar crack-tip enrichment functions are derived from analytical asymptotic displacement fields around a traction free interfacial crack. In this procedure, elements containing a crack tip or strong/weak discontinuities are not required to conform to those geometries. The domain interaction integral approach is also adopted in order to numerically evaluate the mixed-mode stress intensity factors. A number of benchmark tests are simulated to assess the performance of the proposed approach and the results are compared to available reference results.

1. Introduction

According to the huge application of composite materials in many industrial and engineering applications due to their excellent stiffness to weight and strength to weight ratios, the needs of analyzing and modeling of such materials have been of great interest in recent decades. Delamination is one of the most commonly encountered failure modes in composite laminates and can cause severe performance and safety problems, such as stiffness and load bearing capacity reduction and even structural disintegrity.

There are many numerical methods for analysing orthotropic composites, including the boundary element method (BEM), the finite element method (FEM), the finite difference method (FDM), and meshless methods. Although the finite element method is capable of modeling general boundary conditions and complex geometries, the elements associated with cracks must conform to crack faces and remeshing techniques are then required to simulate the crack propagation. This method has fundamental difficulties to reproduce the singular stress field around a crack tip as predicted by the concepts of fracture mechanics.

In contrast, the extended finite element method (XFEM) is specifically designed to enhance the conventional FEM in order to solve problems that exhibit strong and weak discontinuities in material and geometric behavior, while preserving the finite element original advantages. The basis of this method was originally proposed by Belytschko and Black [1], combining the finite element method

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with the concept of partition of unity to improve the FEM deficiencies in modeling discontinuities. In XFEM, elements around a crack are enriched with a discontinuous function and the near-tip asymptotic displacement fields. The major advantage of this method is that the mesh is prepared independent of the existence of any discontinuities.

Asadpoure et al. [2-4] and Mohammadi [5] have extended the method to orthotropic media by deriving new set of orthotropic enrichment functions.

In this research, XFEM is adopted and further improved for modeling interfacial cracks between two orthotropic media by new set of bimaterial orthotropic enrichment functions, completing the earlier research proposed by Esna Ashari and Mohammadi [6]. The new interlaminar crack-tip enrichment functions are derived from analytical asymptotic displacement fields around a traction free interfacial crack. Combined mode I and mode II loading conditions are studied and mixed-mode stress intensity factors (SIFs) are numerically evaluated to determine fracture properties of a problem using the domain form of the contour interaction integral. In order to examine the performance of the proposed approach, two numerical examples are simulated and the results are compared with reference solutions.

2. Extended finite element method

The eXtended Finite Element Method (XFEM) is a way to facilitate modeling strong and weak discontinuities within finite elements by enriching the classical finite element displacement approximation using the framework of partition of unity. This allowed the method to model the discontinuity independent of the finite elements, without explicitly meshing the crack surfaces.
In order to model crack surfaces and crack tips in the extended finite element method, the approximate displacement function $\boldsymbol{u}^h$ can be expressed as

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_{\substack{I \\ n_I \in \boldsymbol{N}}} \phi_I(\boldsymbol{x})\boldsymbol{u}_I + \sum_{\substack{J \\ n_J \in \boldsymbol{N}^H}} \boldsymbol{a}_J \phi_J(\boldsymbol{x}) H(\boldsymbol{x}) + \sum_{\substack{k \\ n_k \in N^F}} \phi_k(\boldsymbol{x}) \left(\sum_l \boldsymbol{b}_k^l F_l(\boldsymbol{x}) \right) + \sum_{\substack{r \\ n_r \in N^\chi}} \boldsymbol{c}_r \phi_r(\boldsymbol{x}) \chi_r(\boldsymbol{x}) \quad (1)$$

where N^H is the set of nodes that have crack face (but not crack-tip) in their support domain, $\boldsymbol{a}_J$ is the vector of additional degrees of nodal freedom and is applied in modeling crack faces (not crack-tips), $H(\boldsymbol{x})$ is the heaviside function used to express the discontinuity of displacement across a crack, N^F is the set of nodes associated with the crack-tip in its influence domain, $\boldsymbol{b}_k^l$ is the vector of additional degrees of nodal freedom for modeling crack-tips, $F_l(\boldsymbol{x})$ are crack-tip enrichment functions, N^χ is the set of nodes that have weak discontinuity, $\boldsymbol{c}_r$ is the vector of additional degrees of nodal freedom for modeling weak discontinuity interfaces and $\chi_r(\boldsymbol{x})$ is the enrichment function used for modeling weak discontinuities.

In equation (1), the first term is the classical finite element approximation, the second term is the enriched approximation related to crack surfaces, the third term is the enriched approximation for modeling crack tips, while the last part is the enriched approximation used for modeling weak discontinuities. These three types of enriched nodes in a finite element modeling of an interface crack are depicted in figure 1. Other nodes and their associated classical finite element degrees of freedom are not affected by the presence of the crack.

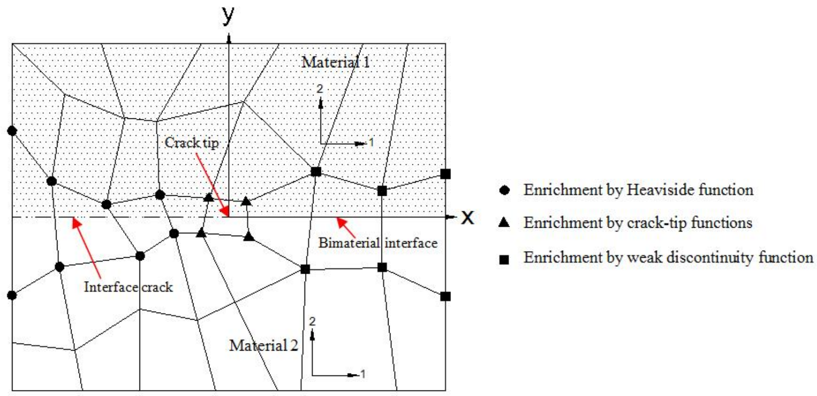

Figure 1. Node selection for enrichment; nodes enriched with crack-tip, heaviside and weak discontinuity functions are marked by triangles, circles and squares, respectively [7].

3. Orthotropic interface enrichments

In order to enhance the accuracy of approximation around an orthotropic bimaterial interface crack tip, the following asymptotic crack-tip functions are extracted from the general crack tip displacement fields

$$\begin{aligned}\{F_l(r,\theta)\}_{l=1}^{8} = \Big[& e^{-\varepsilon\theta_l}\cos\left(\varepsilon\ln(r_l)+\frac{\theta_l}{2}\right)\sqrt{r_l},\ e^{-\varepsilon\theta_l}\sin\left(\varepsilon\ln(r_l)+\frac{\theta_l}{2}\right)\sqrt{r_l}, \\ & e^{\varepsilon\theta_l}\cos\left(\varepsilon\ln(r_l)-\frac{\theta_l}{2}\right)\sqrt{r_l},\ e^{\varepsilon\theta_l}\sin\left(\varepsilon\ln(r_l)-\frac{\theta_l}{2}\right)\sqrt{r_l}, \\ & e^{-\varepsilon\theta_s}\cos\left(\varepsilon\ln(r_s)+\frac{\theta_s}{2}\right)\sqrt{r_s},\ e^{-\varepsilon\theta_s}\sin\left(\varepsilon\ln(r_s)+\frac{\theta_s}{2}\right)\sqrt{r_s}, \\ & e^{\varepsilon\theta_s}\cos\left(\varepsilon\ln(r_s)-\frac{\theta_s}{2}\right)\sqrt{r_s},\ e^{\varepsilon\theta_s}\sin\left(\varepsilon\ln(r_s)-\frac{\theta_s}{2}\right)\sqrt{r_s}\Big]\end{aligned} \tag{2}$$

where θ_l, θ_s, r_l, r_s are defined in terms of the roots of the governing characteristic equations.

These enrichment functions span the analytical asymptotic displacement fields for a traction free interfacial crack.

4. Evaluation of stress intensity factors

The stress intensity factor (SIF) is one of the basic concepts of fracture mechanics to measure the intensity of crack-tip fields and to assess the stability of an existing crack. In this study, the domain integral method, is utilized to evaluate mixed-mode stress intensity factors for an interfacial crack between two orthotropic materials.

The well-known path-independent J integral for a cracked body is defined

$$J = \int_{\Gamma}\left(W\delta_{1j} - \sigma_{ij}\frac{\partial u_j}{\partial x_1} - \right) n_j d\Gamma \tag{3}$$

where Γ is an arbitrary contour surrounding the crack-tip (figure 2), W is the strain energy density, defined by $W = (1/2)\sigma_{ij}\varepsilon_{ij}$ for linear-elastic materials, and n_j is the j^{th} component of the outward unit normal to Γ. This contour integral can be reformulated into the equivalent domain integral.

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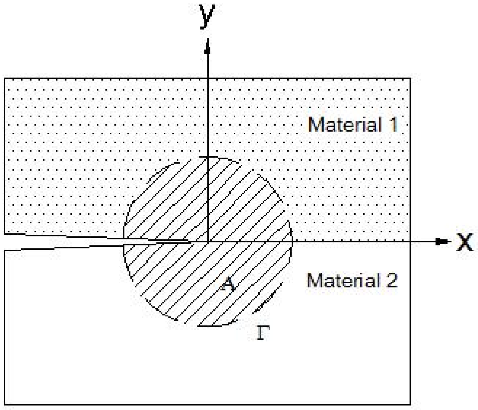

Figure 2. The contour and its interior area, A.

In the interaction integral method, auxiliary fields are introduced and superimposed onto the actual fields to satisfy the boundary value problem (equilibrium equation and traction-free boundary condition on crack surfaces) in order to extract the mixed-mode stress intensity factors. One of the choices for the auxiliary state is the displacement and stress fields in the vicinity of the interfacial crack tip.

The J^S integral for the sum of the two states can be defined as:

$$J^S = J + J^{aux} + M \tag{4}$$

where J and J^{aux} are associated with the actual and auxiliary states, respectively, and M is the interaction integral, defined as

$$M = \int_A \left[\sigma_{ij} u_{i,1}^{aux} + \sigma_{ij}^{aux} u_{i,1} - W^{(1,2)} \delta_{1j}\right] q_{,j} dA \tag{5}$$

For linear elastic condition, $W^{(1,2)}$ is defined as

$$W^{(1,2)} = \frac{1}{2}(\sigma_{ij}\varepsilon_{ij}^{aux} + \sigma_{ij}^{aux}\varepsilon_{ij}) \tag{6}$$

where superscript ^{aux} stands for the auxiliary state. The M integral shares the same path-independent property of J integral and can be utilized to determine the stress intensity factors of the present orthotropic bimaterial problem from the M integral. As a result, it can be calculated away from the crack tip where the finite element solution is more accurate.

5. Numerical examples

5.1. Central crack in an infinite bimaterial orthotropic plate

In this example, the stability of a crack in the interface of two orthotropic materials, as depicted in figure 3, is studied; The infinite plate is subjected to a remote unit tensile loading σ_{22}^0, with the plane strain condition. The material properties of the T300-5208 graphite epoxy are defined as:

$E_T = E_Z = 10.8 \;\; GPa$, $E_L = 137 \;\; GPa$

$G_{ZL} = G_{TL} = 5.65 \;\; GPa$, $G_{ZT} = 3.36 \;\; GPa$

$\nu_{ZL} = \nu_{TL} = 0.238$, $\nu_{TZ} = 3.36$

where *L, T, Z* are longitudinal, transverse and through the thickness directions, respectively. The present example is for a [90º/0º] bimaterial block. The fiber direction in the 90º lamina is along the x_1

axis while for the 0º lamina, the fiber direction lies along the x_3 axis (out of plane direction). The dimensions of the plate are:

$$\frac{W}{a} = \frac{h}{a}, \quad a = 1m$$

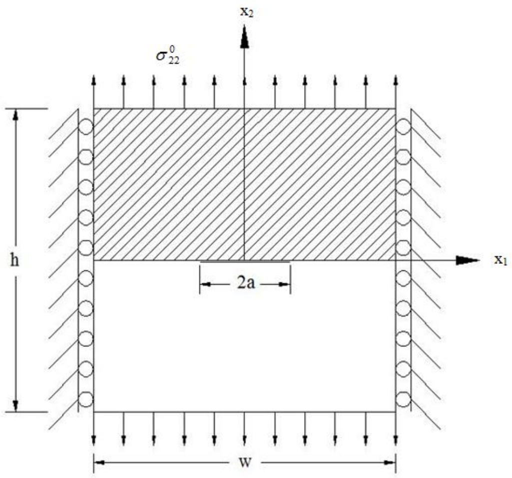

Figure 3. An interfacial crack between two orthotropic materials.

Only one half of the problem is modeled due to symmetry along the x_2 axis. A finite element model with 250 elements and 268 nodes, is employed (figure 4) and the crack tip is modeled with new orthotropic enrichment functions.

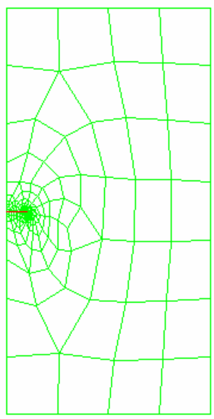

Figure 4. Unstructured FEM model.

To determine the accuracy of the approach, comparisons are made with the exact solution of an infinite aniotropic bimaterial block provided by Qu and Bassani [8], and another investigation by Chow and Atluri [9], based on standard eight noded quarter-points elements and using both the mutual integral method and the extrapolation technique.

Table 1 depicts the results of stress intensity factors obtained by different methods and the extent of error with respect to the analytical solution. Clearly, very accurate results are obtained by new XFEM formulation.

Table 1. Stress intensity factors obtained by different methods compared to the analytical solution by

Qu and Bassani [8].

Method		Number of elements	Number of nodes	Error (K_I)	Error (K_{II})
Chow and Atluri [9]	Mutual Integral	216	679	0.6	0.1
	Mutual Integral	72	237	0.7	13.6
	Extrapolation	72	237	9.5	6.9
	Extrapolation	216	679	13.1	2.3
Present method	XFEM	250	268	0.051	0.824

5.2. Orthotropic double cantilever beam

In this example, XFEM is employed to determine the strain energy release rates for interface cracks in a double cantilever beam (DCB) test with orthotropic materials. A bilayer specimen composed of two homogeneous elastic layers, both of thickness H, with a crack length a is considered, as shown in figure 5. The results of the present simulation are compared with the results reported by Ang et al. [10].

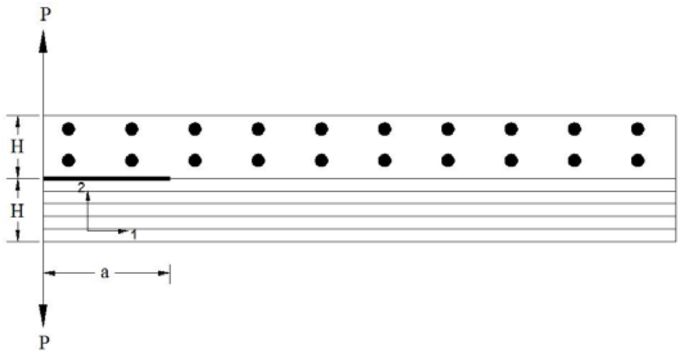

Figure 5. A bilayer orthotropic DCB specimen.

In the lower layer, the fiber directions are along the horizontal direction and perpendicular to the applied load P, with E_1 and E_2 as the Young's moduli in the first and second principal material axes; G_{12} as the in-plane shear modulus and ν_{12} is the Poisson's ratio. The fiber direction is along the out of plane direction in the top layer and it is treated as isotropic in plane with E_2 and ν_{12} as the Young's modulus and the Poisson's ratio, respectively. The mechanical properties of the bottom layer are defined in terms of two parameters η_1 and η_2, which are related to the purely imaginary roots of the characteristic equation of the orthotropic material as: [10]

$$\eta_1\eta_2 = \left(\frac{E_1}{E_2}\right)^{\frac{1}{2}}, \quad \eta_1 + \eta_2 = \sqrt{2\left[\left(\frac{E_1}{E_2}\right)^{\frac{1}{2}} + \frac{E_1}{2G_{12}} - \nu_{12}\right]} \tag{7}$$

In numerical models, the relative crack sizes are $\frac{\mathrm{a}}{H} = 2,4,6$ and $\eta_1 = 0.8, \eta_2 = 2$.

The strain energy release rates are computed for each numerical model. The reference results for strain energy release rates were normalized using G_0:

$$G_0 = \frac{12P^2a^2}{E_1\mathrm{H}^3} \tag{8}$$

Adaptive structured finite element models, depicted in figure 6, with 663 elements and 720 nodes, are employed for different crack lengths.

Table 2 compare the results of normalized strain energy release rates obtained by the present XFEM models using bimaterial orthotropic enrichments and the values reported by Ang et al. [10], which was based on the boundary element method. Similar results, obtained by Ang et al. [10], are illustrated in figure 7, where the results of present XFEM simulations are marked by circles. Very close agreements are observed between the XFEM and reference results.

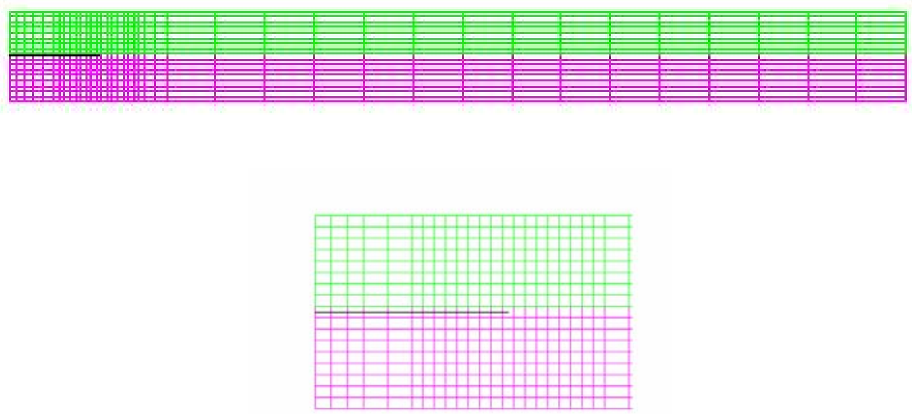

Figure 6. FEM discretization of orthotropic DCB problem.

Table 2. Comparison of the values of normalized strain energy release rates for different crack lengths, obtained from XFEM and the boundary element method (case 1: $\eta_1 = 0.8$ and $\eta_2 = 2$).

$\frac{a}{H}$	$\frac{r_d}{a}$	$\frac{G}{G_0}$	Boundary element method. [10]
2	0.33	3.2996	3.35
4	0.17	2.4768	2.5
6	0.125	2.2313	2.2

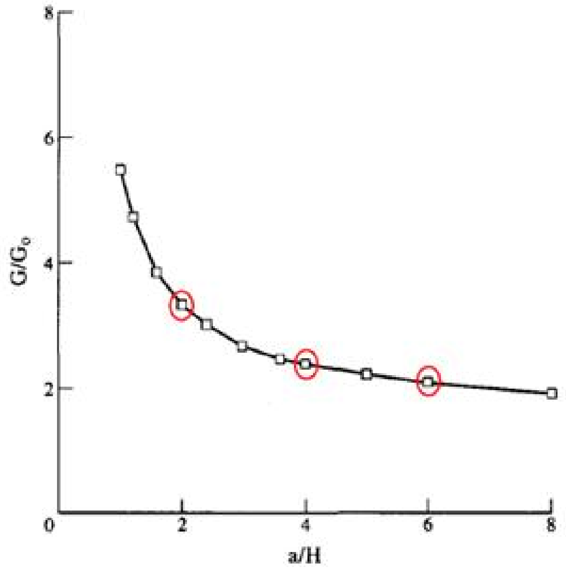

Figure 7. Variations of $\frac{G}{G_0}$ with $\frac{\mathrm{a}}{\mathrm{H}}$ for the DCB specimen ($\eta_1 = 0.8$, $\eta_2 = 2$) using boundary element method [10], compared with XFEM results (circles).

6. Conclusions

The problem of cracks that lie at the interface of two elastically homogeneous orthotropic materials was studied. The extended finite element method (XFEM) was adopted for modeling the interface crack and analyzing the domain numerically. New bimaterial orthotropic crack tip enrichment functions are extracted from the analytical solution in the vicinity of interfacial crack tips. Mixed-mode stress intensity factors and energy release rates for bimaterial interfacial cracks were numerically evaluated using the domain form of the interaction integral. The results obtained by the present method were compared with reference solutions and exhibited close agreement. The combined set of inplane and interlaminar enrichments are expected to allow for a full fracture analysis of layered orthotropic composites by XFEM in future studies.

7. References

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