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Modeling Interface Crack between Orthotropic and Isotropic Materials Using Extended Finite Element Method

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Abstract

In this research, an extended finite element method has been proposed for modeling an interface crack between orthotropic and isotropic materials. To achieve this aim, a discontinuous function and two-dimensional asymptotic crack-tip displacement fields are used in classical finite element approximation enriched with framework of partition of unity. It allows modeling interface crack by finite element method with no explicitly meshing of crack surfaces. Combined mode I and mode II loading conditions are studied. The mixed-mode stress intensity factors (SIFs) are obtained by means of the domain form of the interaction integral (M-integral). An example is provided in this paper and the accuracy of the results is discussed by comparison with other methods.

Keywords: *Extended finite element method; Interface crack; Orthotropic; Isotropic.*

1 Introduction

Advances in material performance, depends on the ability to develop new multi-phase materials. Their performance is controlled not only by the properties of individual phases but also by the strength of their interfaces. Important applications include metal/composite and metal/polymer bimaterial combinations which means isotropic/orthotropic bimaterial combinations. So one of the most important fields of study is the problem of an interface crack between orthotropic and isotropic materials.

There are many numerical methods for analysing interface cracks. The finite element method (FEM) is one of the most popular methods in this field due to capability of modeling general boundary conditions and complex geometries. Beside all advantages, there are some drawbacks in modeling interface cracks by this method. The elements associated with cracks must conform to crack faces and remeshing techniques are then required to simulate the crack propagation.

The extended finite element method (XFEM) (Belytschko and Black, 1999) is specifically designed to improve the conventional FEM in order to solve problems that exhibit strong and weak discontinuities in material and geometric behaviour by combining the finite element method with the concept of partition of unity. In XFEM, elements around a crack are enriched with a discontinuous function and the near-tip asymptotic displacement fields. The major advantage of this method is that the mesh is prepared independent of the existence of any discontinuities. This method was improved in different fields such as modeling crack growth (Möes et al., 1999) and three-dimensional crack modeling (Sukumar et al., 2000). Recently, the method was extended to orthotropic media by deriving a set of orthotropic enrichment functions based on the analytical solutions (Asadpoure and Mohammadi, 2007; Mohammadi, 2008).

In this research, XFEM has been utilized for modeling an interface crack between orthotropic and isotropic materials by new set of bimaterial orthotropic enrichment functions. The new bimaterial orthotropic enrichment functions can be used in both orthotropic and isotropic media. In order to stress analysis of an interface crack between orthotropic and isotropic materials, a discontinuous function and two-dimensional bimaterial orthotropic enrichment functions are used in classical finite element approximation enriched with framework of partition of unity. Combined mode I and mode II loading conditions are studied and mixed-mode stress intensity factors (SIFs) are obtained by means of the domain form of the interaction integral. A numerical example is provided in this paper and the results are compared with reference solutions.

2 Extended finite element method

The extended finite element method (XFEM), is a way to simplify the modeling of continua containing several strong and weak discontinuities by enriching the classical finite element displacement approximation using the framework of partition of unity. This allowed the method to model the discontinuity independent of the finite elements.

In order to model crack surfaces and crack tips in the extended finite element method, the approximate displacement function $\boldsymbol{u}^h$ can be expressed as

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_{\substack{I \\ n_I \in \boldsymbol{N}}} \phi_I(\boldsymbol{x})\boldsymbol{u}_I + \sum_{\substack{J \\ n_J \in \boldsymbol{N}^g}} \phi_J(\boldsymbol{x}) H(\boldsymbol{x}) \boldsymbol{b}_J + \sum_{k \in \boldsymbol{K}^1} \phi_K(\boldsymbol{x}) \left(\sum_l F_l^1(\boldsymbol{x}) \boldsymbol{c}_k^{l1} \right) + \sum_{k \in \boldsymbol{K}^2} \phi_K(\boldsymbol{x}) \left(\sum_l F_l^2(\boldsymbol{x}) \boldsymbol{c}_k^{l2} \right) \quad (1)$$

where $H(\boldsymbol{x})$ is the heaviside function used to express the discontinuity of displacement across a crack, $\boldsymbol{b}_J$ is the vector of additional degrees of freedom which are related to the modeling of crack faces (not crack-tips), N^g is the set of nodes that have crack faces (but not crack-tip) in their support domain, $F_l^i(x), (i = 1,2)$ are crack-tip enrichment functions, $\boldsymbol{c}_k$ is the vector of additional degrees of freedom for modeling crack-tips and $\boldsymbol{K}^1$ and $\boldsymbol{K}^2$ are the sets of nodes associated with crack-tip 1 and 2 in their influence domain, respectively.

In equation (1), the first term is the classical finite element approximation, the second term is the enriched approximation related to crack surfaces and the third term is the enriched approximation for modeling crack tips. These three types of enriched nodes in a finite element modeling of an interface crack are shown in figure 1.

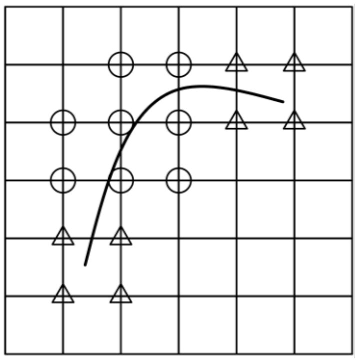

Figure 1. Node selection for enrichment; nodes enriched with crack-tip and heaviside functions are marked by triangles and circles, respectively (Asadpoure et al., 2007).

3 Orthotropic interface crack-tip enrichments

In order to model interface cracks between orthotropic and isotropic materials within the XFEM setting, the following asymptotic crack-tip functions are extracted from the general interfacial crack tip (between two orthotropic materials) displacement fields and modified as the new orthotropic interface enrichment functions:

$$\{F_l(r,\theta)\}_{l=1}^{8} = \left[e^{-\varepsilon\theta_l} \cos\left(\varepsilon \ln(r_l) + \frac{\theta_l}{2}\right)\sqrt{r_l},\ e^{-\varepsilon\theta_l} \sin\left(\varepsilon \ln(r_l) + \frac{\theta_l}{2}\right)\sqrt{r_l}, \right.$$
$$e^{\varepsilon\theta_l} \cos\left(\varepsilon \ln(r_l) - \frac{\theta_l}{2}\right)\sqrt{r_l},\ e^{\varepsilon\theta_l} \sin\left(\varepsilon \ln(r_l) - \frac{\theta_l}{2}\right)\sqrt{r_l},$$
$$e^{-\varepsilon\theta_s} \cos\left(\varepsilon \ln(r_s) + \frac{\theta_s}{2}\right)\sqrt{r_s},\ e^{-\varepsilon\theta_s} \sin\left(\varepsilon \ln(r_s) + \frac{\theta_s}{2}\right)\sqrt{r_s}, \tag{2}$$
$$\left. e^{\varepsilon\theta_s} \cos\left(\varepsilon \ln(r_s) - \frac{\theta_s}{2}\right)\sqrt{r_s},\ e^{\varepsilon\theta_s} \sin\left(\varepsilon \ln(r_s) - \frac{\theta_s}{2}\right)\sqrt{r_s} \right]$$

where θ_l, θ_s, r_l, r_s are defined in terms of the roots of the governing characteristic equations.

These enrichment functions span the analytical asymptotic displacement fields for a general traction free interfacial crack and can be utilized for modeling an interface crack between orthotropic and isotropic materials.

4 Stress intensity factors (SIFs)

In this study, the domain integral method is utilized to evaluate mixed-mode stress intensity factors for an interfacial crack between orthotropic and isotropic materials.

The two-dimensional path-independent J integral is defined as

$$J = \int_{\Gamma} \left(W\delta_{1j} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} - \right) \ n_j d\Gamma \tag{3}$$

where W is the strain energy density, Γ is an arbitrary contour from the lower crack surface to the upper crack surface, and which encloses the crack tip (figure 2) and n is the outward unit normal vector to Γ. The mixed-mode stress intensity factors can be evaluated using the equivalent domain form of the contour integral.

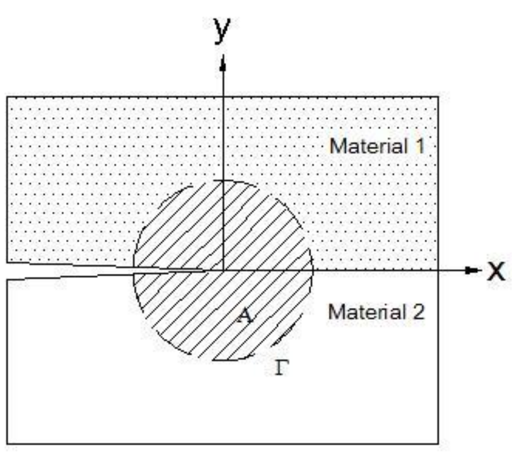

Figure 2. The contour Γ surrounding the interfacial crack-tip.

In the interaction integral method, auxiliary fields are introduced and superimposed onto the actual fields to satisfy the boundary value problem (equilibrium equation and traction-free boundary condition on crack surfaces) in order to extract the mixed-mode stress intensity factors (Sih et al., 1965). One of the choices for the auxiliary state is the displacement and stress fields in the vicinity of the interfacial crack tip.

The J^S integral for the sum of the two states can be defined as:

$$J^S = J + J^{aux} + M \tag{4}$$

where J and J^{aux} are associated with the actual and auxiliary states, respectively, and M is the interaction integral, defined as

$$M = \int_A \left[\sigma_{ij} u_{i,1}^{aux} + \sigma_{ij}^{aux} u_{i,1} - W^{(1,2)} \delta_{1j}\right] q_{,j} \, dA \tag{5}$$

For linear elastic condition, $W^{(1,2)}$ is defined as

$$W^{(1,2)} = \frac{1}{2}(\sigma_{ij}\varepsilon_{ij}^{aux} + \sigma_{ij}^{aux}\varepsilon_{ij}) \tag{6}$$

where superscript ^{aux} stands for the auxiliary state. The M integral can be directly related to the stress intensity factors of the present orthotropic/isotropic bimaterial problem.

5 Numerical example

In this example, an interface crack between isotropic and orthotropic layers of a tensile bimaterial specimen is investigated. The analysis is performed for the bimaterial plate in a plane stress state, subjected to a remote tensile loading and for a range of crack lengths. Figure 3 shows the geometry of the problem.

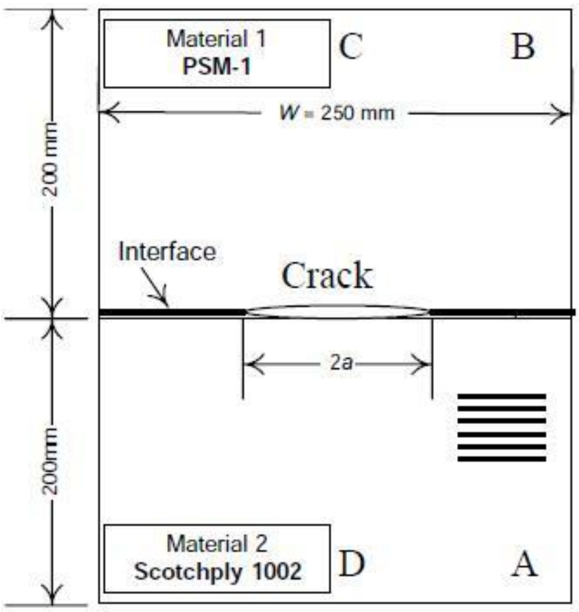

Figure 3. Geometry of a bimatrial plate consisted of isotropic and orthotropic materials.

The layer above the interface is an isotropic PSM-1 material and the material below the interface is composed of orthotropic Scotchply 1002 (a unidirectional glass fiber reinforced composite in epoxy matrix). The material properties of the layers are:

PSM-1:
$E = 2.5Gpa \qquad \mu = 0.91Gpa$

Scotchply 1002
$E_L = 39.3Gpa \qquad E_T = 9.7Gpa \qquad G_{LT} = 3.1Gpa \qquad \nu_{LT} = 0.25$

where L is the direction of fibers and T is the direction transverse to fibers. Fiber orientations are parallel to the interface for orthotropic material.

Since the loading, material and geometry are symmetric, only one half of the plate is considered in the analysis. In figure 3, traction T_y is applied on the upper and lower boundaries; $\tau_{xy} = 0$ on $\overline{AB}$, $T_y = \sigma$ on $\overline{BC}$, $\tau_{xy} = 0, u_x = 0$ on $\overline{CD}$ and $T_y = \sigma$ on $\overline{DA}$.

Adaptive structured finite element models with different numbers of quadrilateral four-noded elements for different crack lengths ($\frac{2a}{w} = 0.2 - 0.6$) are used in this example. Element sizes are substantially smaller in the vicinity of the crack. Figure 4 illustrates the typical mesh utilized for $\frac{2a}{w} = 0.5$.

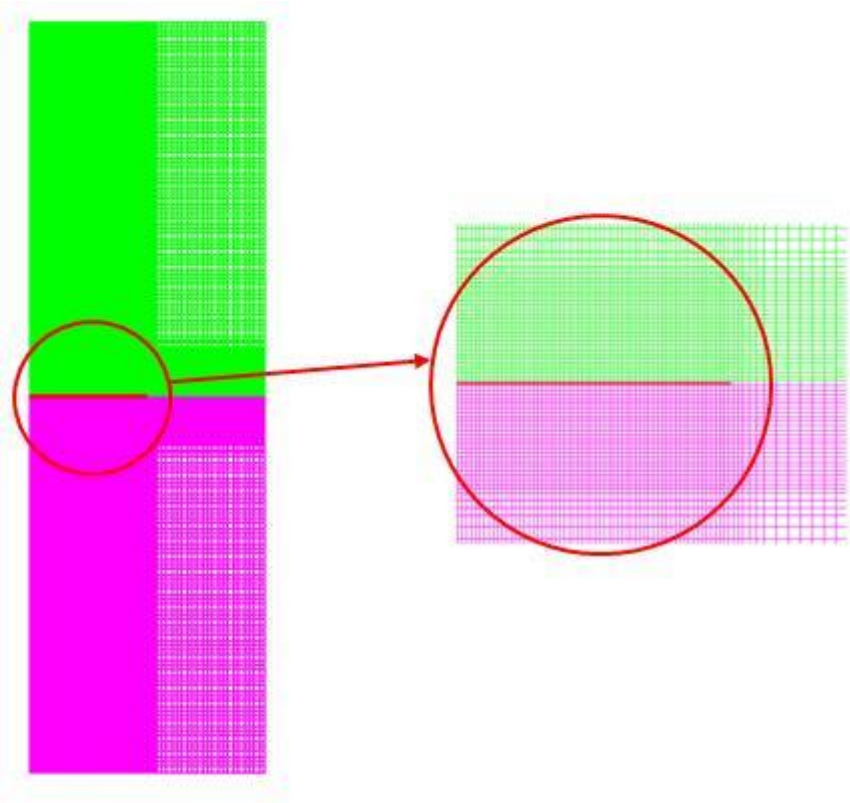

Figure 4. The finite element mesh used for $\frac{2a}{w} = 0.5$.

For each crack length, the complex stress intensity factors (CSIF) are determined and the normalized amplitude of CSIF is calculated as $\frac{|K|}{\sigma\sqrt{\pi a}}$, where $|K| = \sqrt{K_1^2 + K_2^2}$ and σ is the applied far field stress. Table 1 indicates the results of XFEM simulations for different crack lengths based on the new orthotropic enrichment functions. Figure 5 illustrates the normalized complex stress intensity factors with respect to the normalized crack lengths.

Table 1. Values of normalized amplitude of CSIF for different crack lengths ($|K|/\sigma\sqrt{\pi a}$).

	2a/w				
	0.2	0.3	0.4	0.5	0.6
$\lvert K\rvert/\sigma\sqrt{\pi a}$	0.9927	1.0251	1.0734	1.1435	1.2467

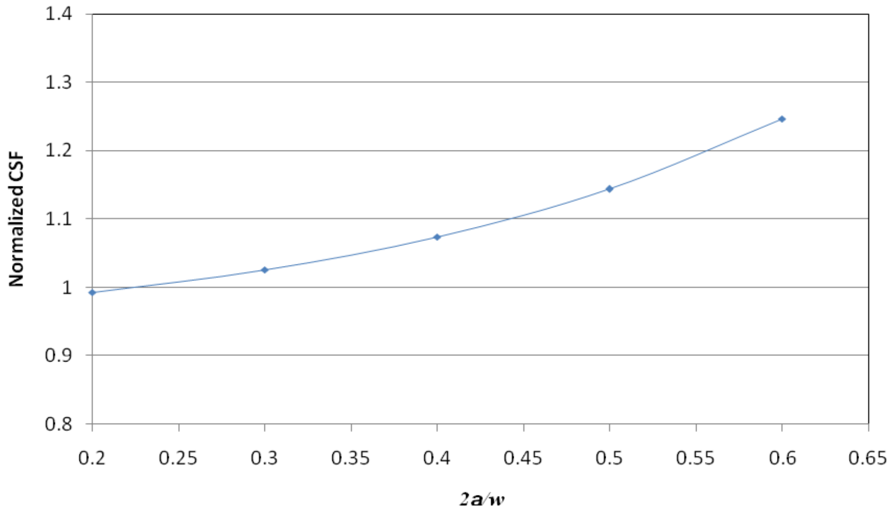

Figure 5. Normalized complex stress intensity factors as a function of normalized crack length.

Another study has been performed on this problem (Shukla et al., 2003) and the normalized complex stress intensity factors were evaluated from experiments and boundary collocation method, as depicted in figure 6. For experimental results, a statistical analysis was performed to obtain the average value and the 95% confidence level. It can be observed that the XFEM numerical estimates of $\frac{|K|}{\sigma\sqrt{\pi a}}$ are in close agreement with reference results (Shukla et al., 2003).

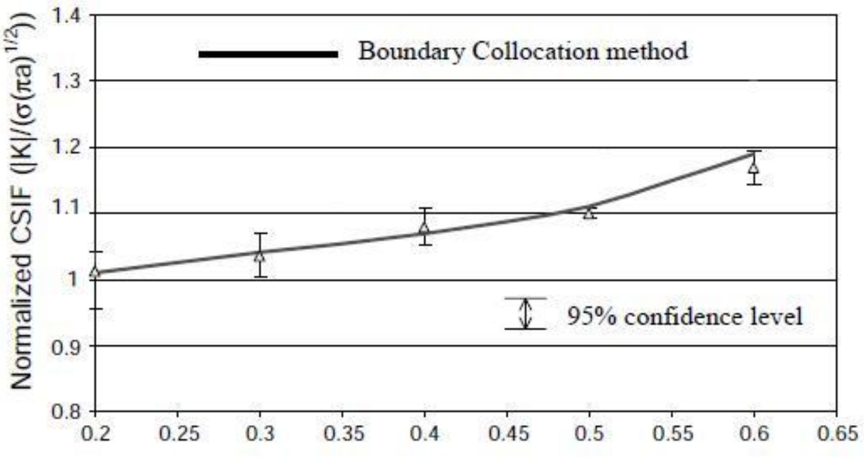

Figure 6. Reference normalized complex stress intensity factors (Shukla et al., 2003)

6 Conclusions

The problem of cracks that lie at the interface of two elastically homogeneous orthotropic and isotropic materials was studied. The extended finite element method (XFEM) was adopted for modeling the interface crack. New bimaterial orthotropic crack tip enrichment functions are extracted from the analytical solution in the vicinity of a general traction free interfacial crack tip between two orthotropic materials. Mixed-mode stress intensity factors were numerically evaluated using the domain form of the interaction integral. The results obtained by the present method were compared with reference solutions and exhibited close agreement. As a result, the analysis of an interface crack between orthotropic and isotropic materials can be performed by only modifying the nodal freedoms near the crack; increasing the accuracy and simplifying the analysis by the conventional FEM.

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