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XFEM DELAMINATION ANALYSIS OF COMPOSITE LAMINATES BY NEW ORTHOTROPIC ENRICHMENT FUNCTIONS

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Summary. The extended finite element method (XFEM), which can simplify and enhance the modeling of strong and weak discontinuities in material and geometric behaviours, is applied to stress analysis of composite laminates containing interlaminar delamination. Newly developed set of orthotropic enrichment functions are utilized in XFEM analysis of linear elastic fracture mechanics of layered composites. Interlaminar crack-tip enrichment functions are derived as those that span the analytical asymptotic displacement fields for a traction free interfacial crack. Combined mode I and mode II loading conditions are studied. The domain interaction integral approach is also adopted in order to determine the mode I and II stress intensity factors.

1 INTRODUCTION

Composite materials, which are being increasingly used in various high performance systems, are vulnerable to interlaminar cracking. Delamination is one of the most common types of damage in laminated composites and can cause severe performance and safety problems, such as stiffness and load bearing capacity reduction and even structural disintegrity.

In earlier simulations, interface elements and continuum based finite element method were used to analyse partially delaminated layered composites. The finite element method, however, has had fundamental difficulties to reproduce the singular stress field around a crack tip as predicted by the concepts of fracture mechanics.

In contrast, the extended finite element method (XFEM) is specifically designed to enhance the conventional FEM in order to solve problems that exhibit strong and weak discontinuities in material and geometric behaviours. This method was originally proposed by Belytschko and Black[1], improved by Möes et al.[2] and further developed by Sukumar et al.[3], among the others. Sukumar et al.[4], applied XFEM to stress analysis of structures containing interface cracks between dissimilar isotropic materials and Nagashima et al.[5] modeled interlaminar delaminations of composite laminates by the near-tip functions for homogeneous isotropic cracks in order to examin the behaviour of orthotropic composite materials. Recently, Asadpoure et al.[6] and Mohammadi[7] have extended the method to orthotropic media by deriving new sets of orthotropic enrichment functions.

In this research, interfacial cracks between two orthotropic media are studied (Figure 1)and new set of orthotropic enrichment functions are proposed. The mixed-mode (complex) stress intensity factors for orthotropic bimaterial interfacial cracks are numerically evaluated using the domain form of the interaction integral. The new proposed enrichment functions are reduced to available isotropic bimaterial interface enrichment functions, proposed earlier by Nagashima et al.[5]. The combined set of inplane and interlaminar enrichments are expected to allow for a full fracture analysis of layered orthotropic composites by XFEM.

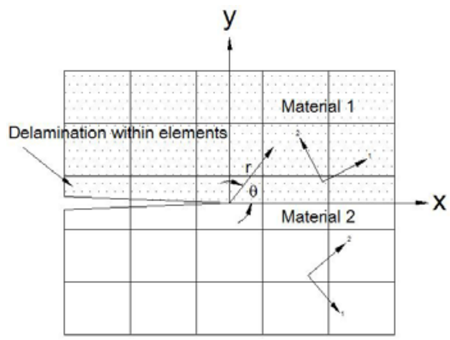

Figure 1: Interfacial crack between two orthotropic materials

2 THE EXTENDED FINITE ELEMENT METHOD

Extended finite element method is used to model cracks and allows the domain to be modelled by finite elements without explicitly meshing the crack surfaces. In XFEM, the approximate displacement function u^h near a delamination can be expressed as:

$$u^h(x) = \sum_{j=1}^{n} N_j(x)u_j + \sum_{h=1}^{m} N_h(x)H(\xi(x))a_h + \sum_{k=1}^{mt_1} N_k(x)\left(\sum_{l=1}^{mf} F_l^1(x)b_k^{l1}\right) + \sum_{k=1}^{mt_2} N_k(x)\left(\sum_{l=1}^{mf} F_l^2(x)b_k^{l2}\right) \tag{1}$$

where N_j is the FEM interpolation function, n is the number FE nodes, m is the set of nodes that have the crack face in their support domain, while mt_i is the set of nodes associated with crack tips i; u_j is the nodal displacement degree of freedom, a_h, b_k^1 and b_k^2 denote the vectors of additional XFEM degrees of freedom, $F_l^i(x)$, $i=1,2$ represent mf crack tip enrichment functions; and $H(x)$ is the Heaviside function used to express the discontinuity of displacement across a crack.

3 ORTHOTROPIC INTERFACE ENRICHMENTS

In order to model interface cracks between two orthotropic materials within the XFEM setting, the following asymptotic crack-tip functions, defined in terms of the local crack tip coordinate system (r,θ), are developed and modified as the new orthotropic interface enrichment functions:

$$F_{\alpha}(r,\theta) = (e^{-\varepsilon\theta_l} cos(\varepsilon \ln r_l + \theta_l/2)\sqrt{r_l}, e^{-\varepsilon\theta_l} sin(\varepsilon \ln(r_l) + \theta_l/2)\sqrt{r_l}, \\ e^{\varepsilon\theta_l} cos(\varepsilon \ln(r_l) - \theta_l/2)\sqrt{r_l}, e^{\varepsilon\theta_l} sin(\varepsilon \ln(r_l) - \theta_l/2)\sqrt{r_l}, e^{-\varepsilon\theta_s} cos(\varepsilon \ln(r_s) + \theta_s/2)\sqrt{r_s}, \\ e^{-\varepsilon\theta_s} sin(\varepsilon \ln(r_s) + \theta_s/2)\sqrt{r_s}, e^{\varepsilon\theta_s} cos(\varepsilon \ln(r_s) - \theta_s/2)\sqrt{r_s}, e^{\varepsilon\theta_s} sin(\varepsilon \ln(r_s) - \theta_s/2)\sqrt{r_s}) \tag{2}$$

where θ_l, θ_s, r_l, r_s are defined in terms of the roots of the governing characteristic equations. Enrichment functions (2) are degenerated to the previously developed isotropic interface enrichment functions, proposed by Sukumar et al.[4]

4 DOMAIN INTER ACTION INTEGRAL METHOD

The well-known path-independent J integral is defined as:

$$J = \int_{\Gamma} (W n_1 - \sigma_{ij} \frac{\partial u_i}{\partial x_1} n_j) d\Gamma \tag{3}$$

where W is the strain energy density and n is the unit outward normal vector(Figure 1). This line integral can be reformulated into the equivalent domain integral. In the interaction integral method, auxiliary fields are introduced and superimposed onto the actual fields satisfying the boundary value problem (Sih et al.[8]). One of the choices for the auxiliary state is the displacement and stress fields in the vicinity of the crack tip. The contour J integral for the sum of the two states can be defined as:

$$J = J^{act} + J^{aux} + M \tag{4}$$

where J^{act} and J^{aux} are associated with the actual and auxiliary states, respectively.

5 NUMERICAL EXAMPLE

The problem of a crack in an infinite tensile orthotropic bimaterial plate of dimentions (30*60) is considered. The material properties are: (All units are kg and cm.)
material 1: E_1=21.84e3, E_2=0.81e3, G_{12}=0.63e3, ν_{12}=21.84.
material 2: E_1=11.84e3, E_2=0.81e3, G_{12}=0.63e3, ν_{12}=21.84.
The evaluated XFEM σ_{yy} and u_y contours are shown in Figure 2. The normalized SIFs are also obtained as:

$$\frac{K_1}{\sigma_{yy}^{\infty}\sqrt{\pi a}} = 3.7434, \frac{K_2}{\sigma_{yy}^{\infty}\sqrt{\pi a}} = 0.0177 \tag{5}$$

where a is the crack length and σ_{yy}^{∞} is the remote traction.

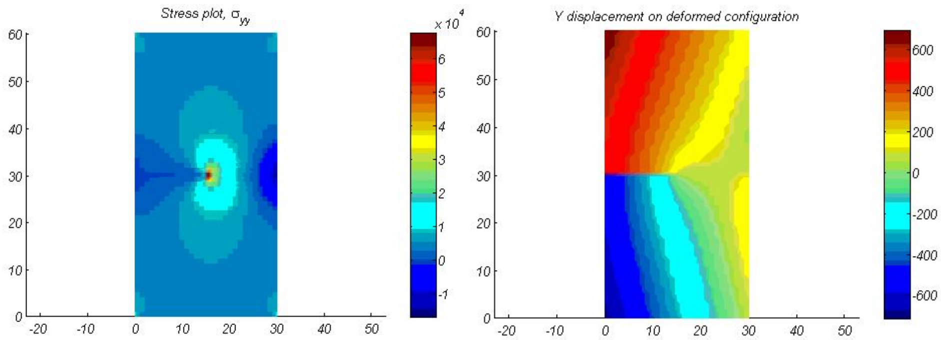

Figure 2: σ_{yy} and y displacement contours

6 CONCLUSIONS

- In this research, new orthotropic enrichment functions for bimaterial interface cracks are proposed. They enhance the capabilities of the extended finite element method for analysis of cracks/delamination at the interface of two elastic orthotropic materials. Interlaminar crack-tip enrichment functions are derived as those that span the analytical asymptotic displacement fields for a traction free interfacial crack. As a result, a delamination analysis can be performed by only modifying the nodal freedoms near the crack; increasing the accuracy and substantially simplifying the delamination analysis of layered composites by the conventional FEM.

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