International Conference on Extended Finite Element Methods - XFEM 2011
June 29 – July 1, 2011, Cardiff, United Kingdom
S. Bordas, B. Karihaloo and P. Kerfriden

ANISOTROPIC/ORTHOTROPIC XFEM FOR FRACTURE ANALYSIS OF STRUCTURES

S. Mohammadi
High Performance Computing Lab, School of Civil Engineering, University of Tehransmoham@ut.ac.ir, http://eng.ut.ac.ir/civ/smohammadi

ABSTRACT

Accurate study of crack stability and propagation in composites has retained its paramount importance for computational community because of vulnerability of these materials to cracking and delamination, which can cause severe performance and safety problems.This paper discusses the recent developments of the extended finite element method (XFEM) for fracture analysis of orthotropic materials. The method is now being extended to other anisotropic applications such as modeling contact and interface, simulation of inclusions and holes, and nano/multiscaleanalyses.This presentation covers all important computational topics in the field of fracture analysis of orthotropic/anisotropic materials, such as fracture analysis of stationary and propagating cracks in single and multilayer orthotropic composites, both in static and dynamic conditions, and for homogenous and functionally graded materials; all important aspects of accurate fracture analysis of composites. The paper also addresses a number of recent developments in closely related topics. The results of anisotropic analysis of dislocations by new anisotropic enrichment functions are presented. This is expected to provide a concrete step forward in realistic simulation of dislocation dynamics.Finally, a perspective for similar anisotropic enrichments in recently developed isogeometric analysis (IGA) is presented. Such anisotropic eXtendedIsoGeometric Analysis (XIGA) has successfully been applied to analysis of various mixed mode fracture analysis of composite laminates.

Key Words: orthotropic materials, crack, extended finite element method (XFEM)

1. Introduction

According to the huge application of composite materials in many industrial and engineering applications, the need for accurate modeling of such materials hasattracted a great interest in recent decades. Delamination is one of the most commonly encountered failure modes in composite laminates and can cause severe performance and safety problems, such as stiffness and load bearing capacity reduction and even structural disintegrity.

There are many numerical methods for analysing orthotropic composites, including the boundary element method, the finite element method (FEM), the finite difference method, and meshless methods. Although FEM is capable of modelling general boundary conditions and complex geometries, the elements associated with cracks must conform to crack faces and remeshing techniques are then required to simulate the crack propagation. This method has fundamental difficulties to reproduce the singular stress field around a crack tip as predicted by fracture mechanics.

In contrast, the extended finite element method (XFEM) is specifically designed to enhance the conventional FEM in order to solve problems that exhibit strong and weak discontinuities in material and geometric behavior, while preserving the finite element original advantages. The basis of this method was originally proposed by Belytschko and Black [1], combining the finite element method with the concept of partition of unity to improve the FEM deficiencies in modeling discontinuities. In XFEM, elements around a crack are enriched with a discontinuous function and the near-tip asymptotic displacement fields. The major advantage of this method is that the mesh is prepared independent of the existence of any discontinuities.

Asadpoure et al. [2-4] and Mohammadi [5] have originally extended the method to orthotropic media by deriving new set of orthotropic enrichment functions, while EsnaAshari and Mohammadi [6] and Motamedi and Mohammadi [7] proposed new crack tip enrichments for orthotropic biomaterials and moving cracks, respectively.

A number of recent developments in closely related topics have also been reported [8-11]. For instance, the results of anisotropic analysis of dislocations by new anisotropic enrichment functions are expected to provide a concrete step forward in realistic simulation of dislocation dynamics [9].

In addition, a perspective for similar anisotropic enrichments in recently developed isogeometric analysis (IGA) is available. Such anisotropic eXtendedIsoGeometric Analysis (XIGA) has successfully been applied to analysis of various mixed mode fracture analysis of composite laminates [10].

In this research, XFEM is presented for modelling interfacial cracks between orthotropic media by new set of bimaterial orthotropic enrichment functions. The new interlaminar crack-tip enrichment functions are derived from analytical asymptotic displacement fields around a traction free interfacial crack. Combined mode I and mode II loading conditions are studied and mixed-mode stress intensity factors (SIFs) are numerically evaluated to determine fracture properties of a problem using the domain form of the contour interaction integral. In order to examine the performance of the proposed approach, two numerical examples are simulated and the results are compared with reference solutions.

2. Extended finite element method

The eXtended Finite Element Method (XFEM) is a way to facilitate modeling strong and weak discontinuities within finite elements by enriching the classical finite element displacement approximation using the framework of partition of unity. This allowed the method to model the discontinuity independent of the finite elements, without explicitly meshing the crack surfaces.

In order to model crack surfaces and crack tips in the extended finite element method, the approximate displacement function $\boldsymbol{u}^h$ can be expressed as

$$\begin{aligned}\mathbf{u}^h(\mathbf{x}) = & \sum_{\substack{I \\ n_I \in \mathbf{N}}} \phi_I(\mathbf{x})\mathbf{u}_I \\ & + \sum_{\substack{J \\ n_J \in \mathbf{N}^H}} \mathbf{a}_J \phi_J(\mathbf{x}) H(\mathbf{x}) + \sum_{\substack{k \\ n_k \in N^F}} \phi_k(\mathbf{x}) \left(\sum_l \mathbf{b}_k^l F_l(\mathbf{x}) \right) + \sum_{\substack{r \\ n_r \in N^{\chi}}} \mathbf{c}_r \phi_r(\mathbf{x}) \chi_r(\mathbf{x})\end{aligned} \tag{1}$$

where N^H is the set of nodes that have crack face (but not crack-tip) in their support

domain, $\boldsymbol{a}_J$ is the vector of additional degrees of nodal freedom and is applied in modelling crack faces (not crack-tips), $H(\boldsymbol{x})$ is the heaviside function used to express the discontinuity of displacement across a crack, N^F is the set of nodes associated with the crack-tip in itsinfluence domain, $\boldsymbol{b}_k^l$ is the vector of additional degrees of nodal freedom for modelling crack-tips, $F_l(\boldsymbol{x})$ are crack-tip enrichment functions, N^{χ} is the set of nodes that have weak discontinuity, $\boldsymbol{c}_r$ is the vector of additional degrees of nodal freedom for modelling weak discontinuity interfaces and $\chi_r(\boldsymbol{x})$ is the enrichment function used for modelling weak discontinuities.
In equation (1), the first term is the classical finite element approximation, the second term is the enriched approximation related to crack surfaces, the third term is the enriched approximation for modelling crack tips, while the last part is the enriched approximation used for modelling weak discontinuities. These three types of enriched nodes in a finite element modelling of an interface crack are depicted in figure 1. Other nodes and their associated classical finite element degrees of freedom are not affected by the presence of the crack.

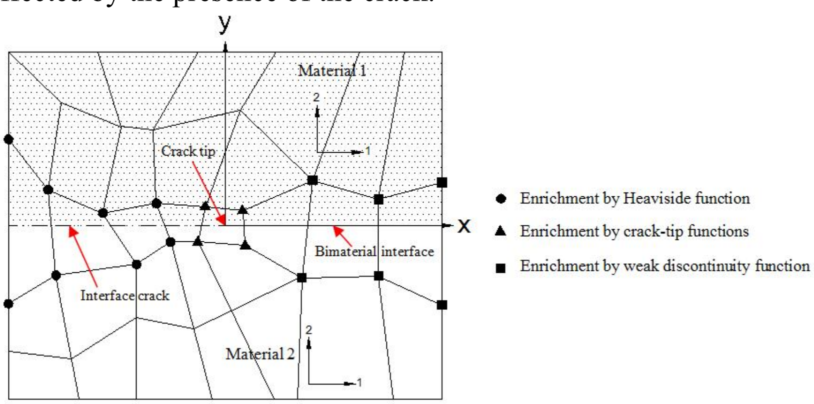

Figure 1. Node selection for enrichment; nodes enriched with crack-tip, heaviside and weak discontinuity functions [6].

3. Orthotropic interface enrichments

In order to enhance the accuracy of approximation around an orthotropic bimaterial interface crack tip, the following asymptotic crack-tip functions are extracted from the general crack tip displacement fields

$$\begin{aligned}
\{F_l(r,\theta)\}_{l=1}^{8} = \Bigg[& e^{-\varepsilon\theta_l}\cos\left(\varepsilon\ln(r_l)+\frac{\theta_l}{2}\right)\sqrt{r_l},\; e^{-\varepsilon\theta_l}\sin\left(\varepsilon\ln(r_l)+\frac{\theta_l}{2}\right)\sqrt{r_l}, \\
& e^{\varepsilon\theta_l}\cos\left(\varepsilon\ln(r_l)-\frac{\theta_l}{2}\right)\sqrt{r_l},\; e^{\varepsilon\theta_l}\sin\left(\varepsilon\ln(r_l)-\frac{\theta_l}{2}\right)\sqrt{r_l}, \\
& e^{-\varepsilon\theta_s}\cos\left(\varepsilon\ln(r_s)+\frac{\theta_s}{2}\right)\sqrt{r_s},\; e^{-\varepsilon\theta_s}\sin\left(\varepsilon\ln(r_s)+\frac{\theta_s}{2}\right)\sqrt{r_s}, \\
& e^{\varepsilon\theta_s}\cos\left(\varepsilon\ln(r_s)-\frac{\theta_s}{2}\right)\sqrt{r_s},\; e^{\varepsilon\theta_s}\sin\left(\varepsilon\ln(r_s)-\frac{\theta_s}{2}\right)\sqrt{r_s} \Bigg]
\end{aligned} \tag{2}$$

where θ_l, θ_s, r_l, r_s are defined in terms of the roots of the governing characteristic equations.

4. Evaluation of stress intensity factors

The stress intensity factor (SIF) is used to measure the intensity of crack-tip fields and to assess the stability of an existing crack,

$$J = \int_{\Gamma} \left(W\delta_{1j} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} - \right) n_j d\Gamma \tag{3}$$

where Γ is an arbitrary contour surrounding the crack-tip (figure 2), W is the strain energy density, defined by $W = (1/2)\sigma_{ij}\varepsilon_{ij}$ for linear-elastic materials, and n_j is the j^{th} component of the outward unit normal to Γ. This contour integral can be reformulated into the equivalent domain integral

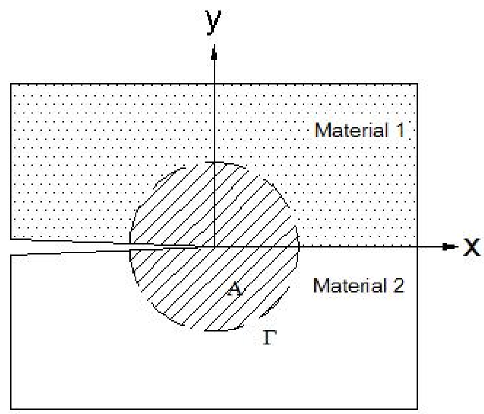

Figure 2. The contour Γ and its interior area, A.

In the interaction integral method, auxiliary fields are introduced and superimposed onto the actual fields to satisfythe boundary value problem (equilibrium equation and traction-free boundary condition on crack surfaces) in order to extract the mixed-mode stress intensity factors. One of the choices for the auxiliary state is the displacement and stress fields in the vicinity of the interfacial crack tip.
The J^S integral for the sum of the two states can be defined as:

$$J^S = J + J^{aux} + M \quad , \quad M = \int_A \left[\sigma_{ij} u_{i,1}^{aux} + \sigma_{ij}^{aux} u_{i,1} - W^{(1,2)} \delta_{1j} \right] q_{,j} dA \tag{4}$$

where $W^{(1,2)} = \frac{1}{2}(\sigma_{ij}\varepsilon_{ij}^{aux} + \sigma_{ij}^{aux}\varepsilon_{ij})$ for linear elastic condition, J and J^{aux} are associated with the actual and auxiliary states, respectively, M is the interaction integral and superscript ^{aux} stands for the auxiliary state. The M integral shares the same path-independent property of J integral and can be utilized to determine the stress intensity factors of the present orthotropic bimaterial problem from the M integral. As a result, it can be calculated away from the crack tip where the finite element solution is more accurate.

5. Numerical example: central crack in an infinite bimaterial orthotropic plate

In this example, the stability of a crack in the interface of two orthotropic materials, as depicted in figure 3, is studied; The infinite plate is subjected to a remote unit tensile loading σ_{22}^{0}, with the plane strain condition. The material properties of the T300-5208 graphite epoxy are defined as:

$E_T = E_Z = 10.8 \quad GPa \ , E_L = 137 \quad GPa$

$G_{ZL} = G_{TL} = 5.65 \quad GPa \ , G_{ZT} = 3.36 \quad GPa$

$\nu_{ZL} = \nu_{TL} = 0.238 \ , \nu_{TZ} = 3.36$

where *L, T, Z* are longitudinal, transverse and through the thickness directions, respectively. The present example is for a [90º/0º] bimaterial block. The fiber direction in the 90º lamina is along the x_1 axis while for the 0º lamina, the fiber direction lies along the x_3 axis (out of plane direction). The dimensions of the plate are: $W = h = \mathrm{a} = 1m$

Only one half of the problem is modelled due to symmetry along the x_2 axis. A finite element model with 250 elements and 268 nodes, is employed (Figure 3) and the crack tip is modelled with new orthotropic enrichment functions.

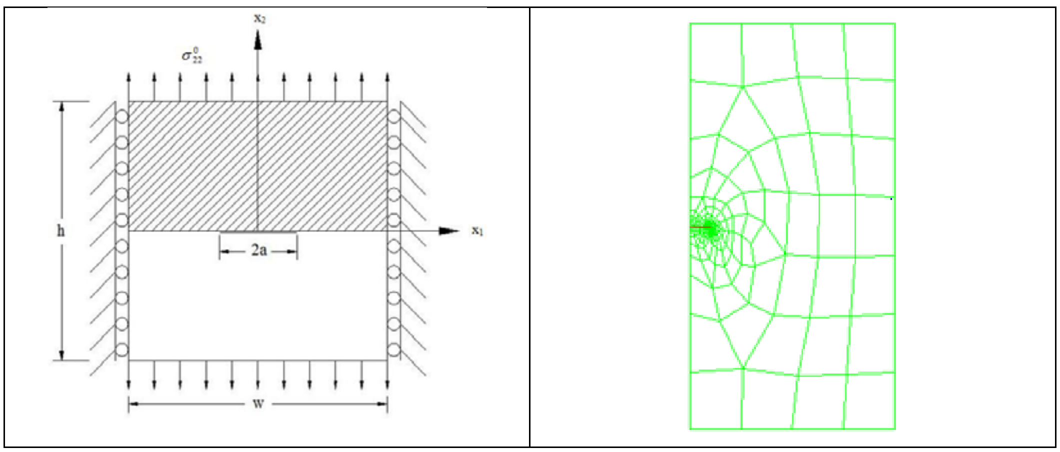

Figure 3. An interfacial crack between two orthotropic materials and the unstructured finite element mesh.

Table 1- Stress intensity factors obtained by different methods compared to the analytical solution by Qu and Bassani [12].

Method		number of		Error	
		elements	nodes	K_I	K_{II}
Chow and Atluri [13]	Mutual Integral	216	679	0.6	0.1
		72	237	0.7	13.6
	Extrapolation	72	237	9.5	6.9
		216	679	13.1	2.3
Present method	XFEM	250	268	0.051	0.824

To determine the accuracy of the approach, comparisons are made with the exact solution of an infinite aniotropicbimaterial block provided by Qu and Bassani [8], and another investigation by Chow and Atluri [9], based on standard eight noded quarter-points elements and using both the mutual integral method and the extrapolation technique.
Table 1 depicts the results of stress intensity factors obtained by different methods and the extent of error with respect to the analytical solution. Clearly, very accurate results are obtained by new XFEM formulation.

Conclusions

The extended finite element method (XFEM) was adopted for modeling the interface cracksthat lie at the interface of two elastically homogeneous orthotropic materials. New bimaterial orthotropic crack tip enrichment functions are extracted from the analytical solution in the vicinity of interfacial crack tips. Mixed-mode stress intensity factors and energy release rates for bimaterial interfacial cracks were numerically evaluated using the domain form of the interaction integral. The results obtained by the present method were compared with reference solutions and exhibited close agreement. Full XFEM fracture analysis of layered orthotropic compositescan now be performed by combined set of inplane and interlaminar enrichments.

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