Analysis of Cracked Orthotropic Media Using the eXtended IsoGeometric Analysis (XIGA)

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1 Introduction

Orthotropic composites are increasingly applied in various engineering applications because of their high specific strength and stiffness characteristics. Accordingly, assessing fracture properties of such materials has attracted interest of many researchers to apply different approaches such as Finite Elements (FE) and the modified crack closure method [1], eXtended Finite Element Method (XFEM) [2] and more recently Element Free Galerkin (EFG) method [3,4] for analysis of cracked orthotropic materials. Further developments have been reported for dynamics and moving cracks in orthotropic media [5,6] and delamination analysis of composites [7].

The IsoGeometric Analysis (IGA) [8] is a newly developed method which is based on application of Non-Uniform Rational B-splines (NURBS) basis function for both the geometric description and the solution field approximation. Its superiorities to the classical Finite Element Method (FEM) such as simple and systematic refinement strategies, exact representation of common and complex engineering shapes, encouraged De Luycker et al. [9] and Ghorashi et al. [10] to extrinsically enrich IGA applying XFEM enrichment functions for analysis of linear elastic fracture mechanics problems. The resulted method, dubbed as the “eXtended IsoGeometric Analysis (XIGA)” [10], benefits from the inherent advantages of both IGA and XFEM, e.g. it is capable of simulating crack propagation problems without remeshing alongside the aforementioned IGA merits.

In this contribution, the XIGA is further develop to analyze cracked orthotropic media by using the orthotropic crack tip enrichment functions, proposed by Asadpoure and Mohammadi [2], in the framework of partition of unity.

2 eXtended IsoGeometric Analysis (XIGA)

IsoGeometric Analysis (IGA) [8] has been recently enriched using superior concepts of eXtended Finite Element Method (XFEM) for analysis of isotropic cracked problems [9,10]. In this method, dubbed as the “eXtended IsoGeometric Analysis (XIGA)” [10], classical IGA space is extrinsically enriched by some additional functions. These functions result from the product of global enrichment functions and some classical NURBS functions. In this case, discontinuous problems can be efficiently analyzed so that the remeshing necessity is vanished in moving discontinuous problems, such as crack propagation simulations. In the XIGA, some basis functions are selected to be enriched by Heaviside and crack tip enrichment functions for modeling crack edges and improving the accuracy of solution field near the crack tip, respectively. In the XIGA, solution field is approximated in the form of
\[
\mathbf{u}_h(\mathbf{\xi}) = \sum_{i=1}^{n_{cp}} R_i(\mathbf{\xi}) \mathbf{u}_i + \sum_{j=1}^{n_{d}} R_j(\mathbf{\xi}) H(\mathbf{\xi}) \mathbf{d}_j + \sum_{k=1}^{n_{ctn}} R_k(\mathbf{\xi}) \left( \sum_{\alpha=1}^{4} Q_{\alpha}(\mathbf{\xi}) \mathbf{c}_{\alpha} \right) \]

(1)

where \( \mathbf{R}(\mathbf{\xi}) \) is the vector of NURBS basis functions, \( \mathbf{d}_j \) is the vector of additional degrees of freedom which are related to the modeling of crack faces, \( \mathbf{c}_{\alpha} \) is the vector of additional degrees of freedom for modeling the crack tip, \( n_{cp} \) is the number of nonzero basis functions for a given knot span, \( n_{d} \) is the number of basis functions that have crack face (but not crack tip) in their support domain and \( n_{ctn} \) is the number of basis functions associated with the crack tip in their influence domain. \( H(\mathbf{\xi}) \) is the Heaviside function, which becomes +1 if \((X, Y)\) (physical coordinates corresponding to the parametric coordinates \(\mathbf{\xi}=(\xi, \eta)\)) is above the crack and -1, otherwise and \( Q_{\alpha}, (\alpha = 1, 2, 3, 4) \) are crack tip enrichment functions which are defined for orthotropic materials in the next section.

### 3 Orthotropic enrichment functions

Asadpoure and Mohammadi [2] proposed the following orthotropic crack tip enrichment functions for XFEM:

\[
\mathbf{Q}(r, \theta) = [Q_1, Q_2, Q_3, Q_4] = \left[ \sqrt{r} \cos \frac{\theta}{2} \sqrt{g_1(\theta)}, \sqrt{r} \cos \frac{\theta}{2} \sqrt{g_2(\theta)}, \sqrt{r} \sin \frac{\theta}{2} \sqrt{g_1(\theta)}, \sqrt{r} \sin \frac{\theta}{2} \sqrt{g_2(\theta)} \right]
\]

(2)

where

\[
\theta_j = \arctan \left( \frac{s_{jx} \sin \theta}{\cos \theta + s_{jy} \sin \theta} \right) \quad j = 1, 2
\]

(3)

\[
g_j(\theta) = \left( \cos \theta + s_{jx} \sin \theta \right)^2 + \left( s_{jy} \sin \theta \right)^2 \quad j = 1, 2
\]

(4)

where \((r, \theta)\) are the local polar coordinates at the crack tip. \(s_{jx}\) and \(s_{jy}\) are real and imaginary parts of characteristic roots \(s_j (j = 1, 2)\), respectively, which are calculated by solving the following characteristic equation,

\[
c_{11}s^4 - 2c_{13}s^3 + (2c_{12} + c_{33})s^2 - 2c_{22}s + c_{22} = 0
\]

(5)

In the above equation, \(c_{ij} (i, j = 1, 2, 3)\) are the components of 2D orthotropic compliance matrix \((\mathbf{\varepsilon} = \mathbf{C} \mathbf{\sigma})\). It is noted that the roots of above equation are always complex or purely imaginary \((s_j = s_{jx} + is_{jy}, \quad j = 1, 2)\) and occur in conjugate pairs as \(s_1, \bar{s}_1\) and \(s_2, \bar{s}_2\).

### 4 Numerical simulation

In order to evaluate the efficiency and validity of the proposed approach, an orthotropic disk with an inclined central crack subjected to double point loads (figure 1(a)) is solved. The material properties and material orthotropy axes are displayed in figure 1(a). 841 control points and 441 elements are used for modeling the problem which are illustrated in figures 1(b) and 1(c), respectively. The order of NURBS functions in both parametric directions \(\xi\) and \(\eta\) are considered three \((p = q = 3)\), all knot vectors are open and uniform without any interior repetition and 2x2 Gauss quadrature and sub-triangles technique with 13 Gauss points in each sub-triangle are used for integration. The Lagrange multiplier method has been adopted for imposition of essential boundary conditions [11]. For determining the
fracture properties, stress intensity factors (SIFs) are obtained by means of the interaction integral method which developed by Kim and Paulino [12].

![Image of an orthotropic disk with an inclined central crack](image)

Figure 1: An orthotropic disk with inclined central crack: (a) Geometry and boundary conditions; (b) Control points; (c) Elements.

This problem has been solved by Kim and Paulino [1], Asadpoure and Mohammadi [2] and Ghorashi et al. [4] before. Table 1 compares the stress intensity factors reported before by those obtained using the present approach for the case of $\alpha = 30^\circ$.

Table 1: Stress intensity factors for an inclined central crack in an orthotropic disk subjected to point loads ($\alpha = 30^\circ$).

<table>
<thead>
<tr>
<th>Method</th>
<th>DOFs</th>
<th>Elements</th>
<th>Cells</th>
<th>$K_I$</th>
<th>$K_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim and Paulino [1] (MCC)</td>
<td>5424</td>
<td>999</td>
<td>-</td>
<td>16.73</td>
<td>11.33</td>
</tr>
<tr>
<td>Kim and Paulino [1] (M- integral)</td>
<td>5424</td>
<td>999</td>
<td>-</td>
<td>16.75</td>
<td>11.38</td>
</tr>
<tr>
<td>Asadpoure and Mohammadi [2]</td>
<td>1960</td>
<td>920</td>
<td>-</td>
<td>17.08</td>
<td>11.65</td>
</tr>
<tr>
<td>Ghorashi et al. [4]</td>
<td>1507</td>
<td>-</td>
<td>641</td>
<td>16.98</td>
<td>11.95</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>1223</td>
<td>441</td>
<td>-</td>
<td>16.68</td>
<td>11.53</td>
</tr>
</tbody>
</table>

XIGA is used for analysis of this problem considering different inclination angles of the crack. The computed values of mixed mode SIFs alongside those of reported by Asadpoure and Mohammadi [2] and Ghorashi et al. [4] are illustrated in figure 2. The results of XIGA are in good agreement with the others. Note that similar to XFEM and EFG method, XIGA enables us to analyze the problem of all crack inclinations applying only one discretization.

5 Conclusion

In this paper, the recently developed XIGA has been further extended to analyze cracked orthotropic media. The orthotropic crack tip enrichment functions, applied in the XFEM, have been successfully adopted in XIGA to impose singular stress fields near the crack tip in the solution field. An orthotropic disk with an inclined central crack has been solved by the proposed approach. Results of mixed-mode stress intensity factors (SIFs) have been compared with the reference results and proved the validity, accuracy and efficiency of the proposed approach.
Figure 2: SIF values corresponding to different central crack angles in the orthotropic disk: (a) mode I SIF; (b) mode II SIF.

References